Science College Congress Nagar, Nagpur
First Unit Test. – 2021-22
Sub: Mathematics and Statistics

Time: 3Hrs) **Class: XII** **Marks: 80**

Instructions: The question paper is divided into **FOUR** sections-

1) **Section A**: Q. no. 1 contains ***eight*** multiple choice type questions carrying **two** marks each. Q. no. 2 contains ***four*** very short answer type of questions carrying ***one*** mark each.
2) **Section B:** Contains ***twelve*** short answer type questions carrying two marks each. (Attempt any ***eight***)
3) **Section C:** contains ***twelve*** short answer type questions carrying ***three*** marks each. (Attempt any ***eight***)
4) **Section D:** Contains ***eight*** long answer type questions carrying four marks each. (Attempt any ***five***)
5) Use of logarithmic table is allowed. Use of calculator is not allowed.
6) Figures to the right indicate full marks.
7) Use of graph paper is not necessary. Only tough sketch is expected.
8) For each MCQ. Correct answer must be written along with its alphabet:

e.g. (a)…./ (b)…./ (c)……/(d)….. Only first attempt will be considered for evaluation.

SECTION – A

Q1) Select and write the most appropriate answer from the given alternatives for each sub-question: [16M]

i) The value of $\left(2tan^{-1}\frac{1}{3}\right)$ is

a) $\frac{15}{27}$ **b)** $\frac{14}{27}$ **c)** $\frac{3}{4}$ **d)** $\frac{13}{27}$

ii) If the angles made by the lines represented by the equation

$ax^2 + 2hxy + by^2 = 0$ with X-axis are α and β, then tan(α+β) is:

a) $\frac{h}{a+b}$ **b)** $\frac{2h}{a-b}$ **c)** $\frac{2h}{a+b}$ **d)** $\frac{h}{a-b}$

iii) if $\bar{a} + \bar{b} = \bar{c}$, $|\bar{a}| = \sqrt{5}$, $|\bar{b}| = \sqrt{2}$, $|\bar{c}| = 3$, then the angle between $\bar{b}$ and $\bar{c}$ is:

a) 30^0 **b)** 45^0 **c)** 60^0 **d)** 90^0

iv) if y = $cot^{-1}\left(\frac{1+6x^2}{x}\right)$, then $\frac{dy}{dx} =$

a) $\frac{1}{1+9x^2} - \frac{1}{1+4x^2}$ **b)** $\frac{3}{1+9x^2} - \frac{2}{1+4x^2}$ **c)** $\frac{6}{1+9x^2}$ **d)** $\frac{12}{1+9x^2}$

v) The Points A(-1, 3, 2), B(-4, 2, -2), and C(5, 5, λ) are collinear, then the value of λ is:

a) 10 **b)** 9 **c)** 8 **d)** 7

vi) $\int \frac{1+cos^2x}{1-cos2x} dx =$

a) $-\frac{1}{2}cotx - \frac{x}{2} + c$ **b)** $\frac{1}{2}cotx - \frac{x}{2} + c$ **c)** $-cotx - \frac{x}{2} + c$ **d)** $cotx - \frac{x}{2} + c$

vii) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1+sinx)^3} dx$ is equal to

a) $\frac{1}{8}$ **b)** $\frac{-1}{8}$ **c)** $\frac{3}{8}$ **d)** $\frac{5}{8}$

vii) the differential equation for $y = e^x(c_1x + c_2)$ is

a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ **b)** $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

c) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - x = 0$ **c)** $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

Q2) Answer the following: **[4M]**

i) If $y = \sin^{-1}x + \cos^{-1}x$, find $\frac{dy}{dx}$

ii) Write the contrapositive of the statement $(p \rightarrow q) \rightarrow p$.

iii) Find the value of $cos^{-1}\left(\cos\frac{9\pi}{4}\right)$.

iv) Form the differential equation by eliminating the arbitrary constants from $y^2 = ax + b$.

SECTION – B

Q3) Attempt any EIGHT of the following: **[16M]**

i) If θ is the acute angle between the lines given by $3x^2 - 4xy + by^2 = 0$ and $\tan\theta = \frac{1}{2}$, find b.

ii) Find the volume of the parallelepiped whose coterminous edges are $2\hat{\imath} - 3\hat{\jmath}$, $\hat{\imath} + \hat{\jmath} - \hat{k}$ and $3\hat{\imath} - \hat{k}$.

iii) Find the general solution of the differential equation

$tany\frac{dy}{dx} = \sin(x + y) - \sin(x - y)$

iv) Evaluate: $\int \sin^3 x \cos^3 x \, dx$

v) Find the direction ratios of a line perpendicular to both the lines whose direction ratios are 3, 2, -1 and 2, 4, -2.

vi) Solve the equation: $tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}tan^{-1}x$, for x >0.

vii) Find the value of 'a' if $\int_2^a (x+1)dx = \frac{7}{2}$

viii) Find the inverse of the matrix A = $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$ by adjoint method.

ix) Differentiate e^x cos x w. r. t. e^{-x} sin x.

x) The side of a square is increasing at the rate of 0.5cm/sec. Find the rate of increase of the perimeter when the side of the square is 10 cm long.

xi) The p.m.f. of a random variable X is as follows:

P(X = 0) = $5k^2$, P(X = 1) = 1 – 4k, P(X = 2) = 1 – 2k and P(X = x) = 0 for any other value of X. Find k.

xii) A fair coin is tossed 6 times. Find the probability of getting heads 4 times.

SECTION – C

Q4) Attempt any EIGHT of the following questions: [24]

i) If tan θ + sec θ = $\sqrt{3}$, find the general value of θ.

ii) Show that the simplified form of (p ^ q ^ ~r) v (r ^ p ^ q) v (~p v q) is q v ~p.

iii) Form the differential equation by eliminating the arbitrary constants:

y = A cos(log x)+B sin(log x)

iv) if $\hat{\imath} + \hat{\jmath} + \hat{k}$, $\hat{\imath} - \hat{\jmath} + \hat{k}$ and $2\hat{\imath} + 3\hat{\jmath} + m\hat{k}$ are coplanar then find the value of m.

v) Evaluate: $\int \frac{\sin(x-a)}{\sin(x-a)}dx$

vi) Evaluate: $\int_0^1 \frac{dx}{x+\sqrt{1-x^2}}$

vii) Find the equation of the plane passing through the line of intersection of the planes 2x – y + z = 3 and 4x – 3y + 5z = -9 and parallel to the line

$\frac{x+1}{2} = \frac{y+3}{4} = \frac{z-3}{5}$

viii) if $x^p \cdot y^q = (x+y)^{p+q}$, show that $\frac{dy}{dx} = \frac{y}{x}$

ix) At what point does the curve y = 3x – x^2 have slope -5?

x) Evaluate: $\int \frac{2}{(1-x)(1+x^2)}dx$.

xi) Find the mean of number of heads in three tosses of a fair coin.

xii) For the binomial distribution X ~ B(n, p), n = 6 and P(X = 4) = P(X = 2). find p.

SECTION – D

Q5) Attempt any FIVE of the following: **[20M]**

i) If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ then find A^{-1} by adjoint method.

ii) Show that the line $3x + 4y + 5 = 0$ and the lines $(3x + 4y)^2 - 3(4x - 3y)^2 = 0$ form an equilateral triangle.

iii) Find the area enclosed by the parabola $y^2 = x$ and the line $x + y = 2$.

iv) Show that the points with position vectors $\bar{p}$, $\bar{q}$, $\frac{5\bar{p}-3\bar{q}}{2}$ and $\frac{2\bar{p}+\bar{q}}{3}$ are collinear.

v) Maximize $z = 6x + 4y$, subject to $x \leq 2$, $x + y \leq 3$, $-2x + y \leq 1$, $x \geq 0$, $y \geq 0$.

vi) Differentiate $tan^{-1}\left(e^x \frac{x}{\sqrt{1-x^2}}\right)$ w.r.t. $sec^{-1}\left(\frac{1}{2x^2-1}\right)$.

vii) Evaluate: $\int \text{cosec}^3 x \, dx$.

viii) Find the dimension of a right circular cone of maximum volume which can be inscribed in a sphere of radius r.

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