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REAL GASES AS LOW TEMPERATURE GAUGE

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Abstract Almost all text books deal with the Van der waals (vdW) equation and is riddling in literature since 1870. This equation is widely appreciated. Like Boyle's Temperature, we propose Boyle's Volume that offers a condition enforcing vdW equation reduce to volume independent-ideal gas equation. This equation can be explored to calibrate temperature gauge for known pressure.

Keywords Van der Waal's equation, Real gas equation, cubics, equation of state, analytical solution etc.

Introduction

The subject Kinetic Theory of Gases is popular in undergraduate Physics curriculum. Van der Waal's equation (vdW) of state is well familiar. The present paper proposes a view point of Boyles Volume that reduces vdW's equation to ideal gas equation that is volume independent. However, vdW's coefficients *a*, *b* appears that are well-known in the literature. This equation can be used to calibrate thermometer in terms of pressure.

Van der Waals' equation of state for real gas

Van der Waals' (vdW) equation is

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT \quad .....(1)$$

where *P* is the pressure, *V* is the volume, *T* is the absolute temperature, *R* is the gas constant, and *a* and *b* are constants that depend upon the gas.

In the usual situation *a* and *b* are known in the literature. One must watch the units used though. Usually *a* is in liter^2 atm mole^{-2}. Similarly *b* is most often in liter mole^{-2}. Thus it is usually convenient to work in liter-atm and to take *R*=0. 0820578 liter-atm/mole-kelvin. [1-2]

In a problem, if the volume and temperature were known and the pressure were needed, we simply use (1) in form

$$P=\frac{RT}{(V-b)}-\frac{a}{V^2}\ldots\ldots\ldots\ldots (2)$$

Or if the pressure and volume were known and the temperature were needed:

$$T=\frac{1}{R}\left(P+\frac{a}{V^2}\right)(V-b) \quad .....(3)$$

The issue turns really troublesome when pressure and temperature are known and the volume is required to be found. A look at equation (1) shows that *V* occurs in two different places. It is not *linear* in *V*. A closer look reveals it to be cubic in *V*.

Solving cubics is not as simple as solving quadratics. There is a "cubic formula", but it is quite messy and takes a large amount of work.[1]

vdW's equation for the volume is a cubic and cubics may have three roots. If exists, one is guaranteed to be real and hence meaningful; the other two can be either real or complex. We are not concerned with the imaginary root as it is of less physical significance.

van der Waals' reducing to Ideal Gas Equation

Eq (1) has been rearranged

$$\frac{PV}{RT} = 1 + \left(b - \frac{a}{RT}\right)\frac{1}{V} + b^2\frac{1}{V^2} \quad ..(4)$$

Second term on rhs is vanishing for

$$b = \frac{a}{RT}.$$

The temperature corresponding is know as **Boyle's temperature**

$$T_{Boyle} = \frac{a}{Rb}.$$

The higher power of *b* can be ignored for being negligible. At Boyle's temperature, inter-molecular attractive and repulsive effects are counter balanced and vdW's equation reduces to ideal. This has been established fact.

Here, we are proposing a different approach where we can look for alternate perspective in terms of **Boyle's Volume**. We can re-write equation (1) as

$$PV - \left(Pb - \frac{a}{V} + \frac{ab}{V^2}\right) = RT \quad ....(5)$$

Last equation may appear like ideal gas equation $PV = RT$, if the term in the bracket vanishes;

$$X := Pb - \frac{a}{V} + \frac{ab}{V^2} = 0 \quad .....(6)$$

Now we shall probe the conditions and meaning that vanish quadratic equation *X*. A simple re-arrangement gives it a look we usually prefer as;

$$PV^2 - \frac{a}{b}V + a = 0 \quad .....(6a)$$

This equation now has resemblance with $ax^2 + bx + c = 0$ that has roots

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

A simple substitution yields roots

$$V = \frac{\pm a + \sqrt{a^2 - 4Pb^2a}}{2Pb}.$$

These roots can be discussed in two parts viz Real and Complex.

It is the volume that enforce

$$X := Pb - \frac{a}{V} + \frac{ab}{V^2} = 0,$$

in turn reduces vdW's equation to ideal gas equation. It may be termed as Boyle's volume.

We can extend interpretation for Boyle's volume that offers inter-molecular attractive and repulsive effects are counter balanced. Hence the vdW's equation reduces to ideal gas equation.

The volume pair (V_1, V_2) that offers X=0, when substituted in ideal gas equation offers volume elimination in terms of a and b as;

$$PV = RT$$

$$\frac{\pm a + \sqrt{a^2 - 4Pb^2 a}}{2b} = RT \text{ .... (7)}$$

Last equation pair on simplification offer unique expression for pressure as

$$P = -\frac{R^2 T^2}{a} + \frac{RT}{b} \text{..........(8)}$$

This equation in independent of V but instead expressed in terms of a and b. The characteristics trend of ideal gas equation and (8) are found same when tested graphically. This equation is linear in the range of low temperature, wherein it can be explored as low temperature pressure gauge. Its functional behavior is like

$$y = -x^2 + x$$

which appears as inverted parabola, with vertex in 1st quadrant and opening downwards as shown in Fig. 1. At vertex

$$\frac{dy}{dx} = -2x^2 + 1 = 0.$$

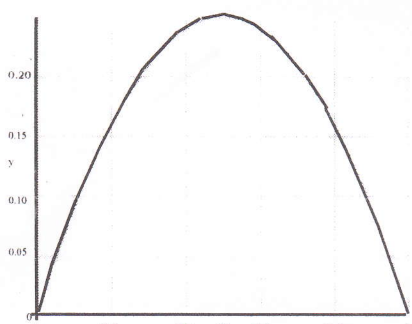

Fig.1 Variation of function $y = -x^2 + x$

Thus, vertex has coordinates $x = 0.5, y = 0.25$. Moreover, $y = x(1 - x)$ suggest $y = 0$ for $x = 0$ and $x = 1$. y may be considered linear for $x < \frac{1}{4}$.

Similarly, maxima for (8) can be worked out from first order derivative as;

$$\frac{dP}{dT} = -2\frac{R^2 T}{a} + \frac{R}{b} = 0$$

i.e. $T = \frac{a}{2Rb}$.

Equation (8) can be factorized to trace x-intercepts (roots) as;

$$P = \frac{RT}{b}\left(1 - \frac{RTb}{a}\right)$$

It is clearly apparent that the roots are

$$T = \left[0, \frac{a}{Rb}\right]$$

We propose the calibration linearity in the range

$$\left[0 - \frac{a}{4Rb}\right]$$

Table 1 depicts the linear temperature range of the gauge for various gas species. However, non-linear region with smart device support can be accommodated in calibration.

T_{max} represents maximum temperature range (in Kelvin) of the gauge.

Molar Gas Constant

$$R = 8.314472 \frac{\text{m}^2\,\text{kg}}{\text{s}^2\,\text{mol K}}.$$

Table 1.

SN	Gas	a	b	$T_{max} = \frac{a}{4Rb}$.
1	He	0.003577	2.41E-05	4.472076
2	Ar	0.135524	3.2E-05	127.2938
3	H_2	0.024785	2.66E-05	28.01186
4	N_2	0.136376	3.86E-05	106.2852
5	O_2	0.137806	3.18E-05	130.1711
6	CO_2	0.365605	4.28E-05	256.6309

Conclusion

Instead of temperature that reduces vdW equation to ideal gas equation we have hunted for volume. This volume termed as Boyle's volume. Boyle's volume offers balanced inter-molecular attractive and repulsive effects. This causes the vdW's equation reduces to ideal gas equation. Boyle's volume combined with ideal gas equation develops volume independent ideal gas equation, but inducts vdW coefficients. The resulting equation in pressure and temperature has a linear range in which it can serve as a temperature gauge. Measuring pressure can help knowing temperature. Adaptability of smart device support can further enlarge calibration region.

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