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Kaluza-Klein Universe in Creation-Field Cosmology

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***ABSTRACT*:** We have studied the Hoyle-Narlikar *C*-field cosmology with Kaluza-Klein space-time. Using methods of Narlikar and Padmanabham (1985), the solutions have been studied when the creation field *C* is a function of time *t* only. The geometrical and physical aspects for models are also studied.

***KEYWORDS*:** Kaluza-Klein universe, Creation field cosmology, Cosmological model of universe.

I. INTRODUCTION

The phenomenon of expanding universe, primordial nucleon-synthesis and the observed isotropy of cosmic microwave background radiation (CMBR) are the three important observations in astronomy which were supposed to be successfully explained by big-bang cosmology based on Einstein's field equations. However, Smoot et al. (1992) revealed that the earlier predictions of the Friedman-Robertson-Walker type of models do not always exactly meet our expectations. Some puzzling results regarding the red shifts from the extra galactic objects continue to contradict the theoretical explanations given from the big bang type of the model. Also, CMBR discovery did not prove it to be a out come of big bang theory. In fact, Narlikar *et al.* (2003) have proved the possibility of non-relic interpretation of CMBR. To explain such phenomenon, many alternative theories have been proposed from time to time. Hoyle (1948), Bondi and Gold (1948) proposed steady state theory in which the universe does not have singular beginning nor an end on the cosmic time scale. Moreover, they have shown that the statistical properties of the large scale features of the universe do not change. Further, the constancy of the mass density has been accounted by continuous creation of matter going on in contrast to the one time infinite and explosive creation of matter at $t = 0$ as in the earlier standard model. But the principle of conservation of matter was violated in this formalism. To overcome this difficulty Hoyle and Narlikar (1966) adopted a field theoretic approach by introducing a mass less and charge less scalar field *C* in the Einstein-Hilbert action to account for the matter creation. In the *C*-field theory introduced by Hoyle and Narlikar there is no big bang type of singularity as in the steady state theory of Bondi and Gold (1948). A solution of Einstein's field equations admitting radiation with negative energy mass less scalar creation fields *C* was obtained by Narlikar and Padmanabhan (1985). The study of Hoyle and Narlikar theory (1963, 1964 and 1966) to the space-time of dimensions more than four was carried out by Chatterjee and Banerjee (2004). RajBali and Tikekar(2007) studied *C*-field cosmology with variable *G* in the flat Friedmann-Robertson-Walker model. Also, *C*-field cosmological models with variable *G* in FRW space–time have been studied by RajBali and Kumawat (2009). The solutions of Einstein's field equations in the presence of creation field have been obtained for Bianchi type universes by Singh and Chaubey (2009). Adhav *et al. (*2010) have studied N-dimensional Bianchi type I Universe in creation field cosmology.

Recently, the cosmology in Kaluza-Klein (1921, 1926) theory has attracted a considerable attention. The Kaluza-Klein theory was introduced to unify Maxwell's theory of electromagnetism

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and Einstein's gravity theory by adding the fifth dimension. Due to its potential function to unify the fundamental interactions, Kaluza-Klein theory has been regarded as a candidate of fundamental theory. Kaluza-Klein theory has been revived in recent years in the modern physics such as in string theory (Polchinski, 1998), in super gravity (Duff *et al*., 1986) and in superstring theories (Green *et al*, 1987). Many of the papers currently published in the context of Kaluza-Klein theory deal with cosmology. Ponce (1988), Chi (1990), Fukui (1993), Liu and Wesson (1994), Coley (1994) have studied Kaluza-Klein cosmological models with different matters. An excellent review of Kaluza-Klein theory and higher dimensional unified theories, have been presented by Overduin and Wesson (1997). Li *et al* (1998) have considered the inflation in Kluza-Klein theory. Wesson and Liu (2001) have investigated the cosmological constant problem in Kaluza-Klein cosmology. Palatnik (2007) constructed Schwarzschild solution for 3 space and n time dimensions in Kaluza-Klein theory. Recently some authors argued that Kaluza-Klein theory can be effective in accounting for dark constituent of the universe (Qiang *et al,* 2005; Chen and Jing, 2009). Adhav *et al*. (2008) have studied string cloud and domain walls with quark matter in N-dimensional Kaluza-Klein cosmological model.

The motivation of this paper is to study concretely the properties of the creation field cosmology in the Kaluza-Klein theory. Here, we have considered a Kaluza-Klein cosmological model in Hoyle and Narlikar *C*-field cosmology with five dimensions. We have assumed that the creation field *C* is a function of time t only i.e. $C(x,t)=C(t)$.

II. HOYLE AND NARLIKAR C-FIELD COSMOLOGY

Introducing a mass less scalar field called as creation field viz. *C*-field, Einstein's field equations are modified. Hoyle and Narlikar (1963, 1964 and 1966) field equations are

$$R_{ij}-\frac{1}{2}g_{ij}R=-8\pi\left({}^{m}T_{ij}+{}^{c}T_{ij}\right), \tag{2.1}$$

where ${}^{m}T_{ij}$ is matter tensor of Einstein theory and ${}^{c}T_{ij}$ is matter tensor due to the *C*-field which is given by

$${}^{c}T_{ij}=-f\left(C_iC_j-\frac{1}{2}g_{ij}C^kC_k\right), \tag{2.2}$$

where $f>0$ and $C_i=\dfrac{\partial C}{\partial x^i}$.

Because of the negative value of $T^{00}\left(T^{00}<0\right)$, the *C*-field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Hence, the energy conservation equation reduces to

$${}^{m}T^{ij}{}_{;j}=-{}^{c}T^{ij}{}_{;j}=fC^iC^j{}_{;j}, \tag{2.3}$$

i.e.: matter creation through non-zero left hand side is possible while conserving the over all energy and momentum. Above equation is similar to

$$mg_{ij}\frac{dx^i}{ds}-C_j=0, \tag{2.4}$$

which implies that the 4-momentum of the created particle is compensated by the 4-momentum of the *C*-field.

In order to maintain the balance, the *C*-field must have negative energy. Further, the *C*-field satisfy the source equation $f\,C^i{}_{;i} = J^i{}_{;i}$, and $J^i = \rho \dfrac{dx^i}{ds} = \rho v^i$, where ρ is homogeneous mass density.

III. METRIC AND FIELD EQUATIONS

Let us consider the Kaluza-Klein type metric as

$$ds^2 = dt^2 - a^2\left(dx^2 + dy^2 + dz^2\right) - b^2 d\psi^2 \quad , \tag{3.1}$$

where *a* and *b* are functions of t only.

We have assumed that creation field *C* is function of time t only., i.e.:

$$C(x,t) = C(t) \quad \text{and} \quad {}^mT^i_j = diag(\rho, -p, -p, -p, -p) \tag{3.2}$$

We have also taken velocity of light is equal to one unit. Also $8\pi G = 1$.

Now, the Hoyle-Narlikar field equations (2.1) for metric (3.1) with the help of equations (2.2) and (3.2) can be written as

$$3\left(\frac{\dot a}{a}\right)^2 + 3\frac{\dot a \dot b}{ab} = 8\pi\left(\rho - \frac{1}{2} f\dot C^2\right) \tag{3.3}$$

$$2\frac{\ddot a}{a} + \frac{\ddot b}{b} + \left(\frac{\dot a}{a}\right)^2 + 2\frac{\dot a \dot b}{ab} = 8\pi\left(-p + \frac{1}{2} f\dot C^2\right) \tag{3.4}$$

$$3\frac{\ddot a}{a} + 3\left(\frac{\dot a}{a}\right)^2 = 8\pi\left(-p + \frac{1}{2} f\dot C^2\right) \tag{3.5}$$

$$\dot\rho + \left(3\frac{\dot a}{a} + \frac{\dot b}{b}\right)(\rho + p) = f\dot C\left[\ddot C + \left(3\frac{\dot a}{a} + \frac{\dot b}{b}\right)\dot C\right], \tag{3.6}$$

where dot $(\cdot)$ indicates the derivative with respect to cosmic time *t*.

Defining $V = a^3 b$ (3.7)

The equation (3.6) can be written in the form

$$\frac{d}{dV}(V\rho) + p = f\dot C(V)\frac{d}{dV}\left[V\dot C(V)\right]. \tag{3.8}$$

In order to obtain a unique solution, one has to specify the rate of creation of matter-energy (at the expense of the negative energy of the *C*-field). Without loss of generality, we assume that the rate of creation of matter energy density is proportional to the strength of the existing *C*-field energy-density. i.e. the rate of creation of matter energy density per unit proper-volume is given by

$$\frac{d}{dV}(V\rho)+p=\alpha^2\dot{C}^2\equiv\alpha^2 g^2(V), \tag{3.9}$$

where α is proportionality constant and we have defined $\dot{C}(V)\equiv g(V)$.

Substituting it in equation (3.8), we get

$$\frac{d}{dV}(V\rho)+p=fg(V)\frac{d}{dV}(Vg). \tag{3.10}$$

Comparing right hand sides of equations (3.9) and (3.10), we get

$$g(V)\frac{d}{dV}(gV)=\frac{\alpha^2}{f}g^2(V). \tag{3.11}$$

Integrating, which gives

$$g(V)=BV^{\left(\frac{\alpha^2}{f}-1\right)}, \tag{3.12}$$

where B is arbitrary constant of integration.

We consider the equation of state of matter as

$$p=\gamma\rho \quad \text{where} \quad 0\leq\gamma\leq 1 . \tag{3.13}$$

Substituting equations (3.12) and (3.13) in the equation (3.11), we get

$$\frac{d}{dV}(V\rho)+\gamma\rho=\alpha^2 B^2 V^{2\left(\frac{\alpha^2}{f}-1\right)}. \tag{3.14}$$

Which further yields

$$\rho=\frac{\alpha^2 B^2}{\left(2\frac{\alpha^2}{f}-1+\gamma\right)}V^{2\left(\frac{\alpha^2}{f}-1\right)} \tag{3.15}$$

Subtracting equation (3.4) from equation (3.5), we get

$$\frac{d}{dt}\left(\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right)+\left(\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right)\frac{\dot{V}}{V}=0 \tag{3.16}$$

Integrating, we get,

$$\frac{a}{b}=d\quad\exp\left(x\int\frac{dt}{V}\right), \tag{3.17}$$

where *d* and *x* are constants of integration.

Using equation (3.7) and equation (3.17) , *a* and *b* can explicitly be written as

$$a = D_1 V^{1/4} \exp\left(X_1 \int \frac{dt}{V} \right), \tag{3.18}$$

$$b = D_2 V^{1/4} \exp\left(X_2 \int \frac{dt}{V} \right) \tag{3.19}$$

where the relations $D_1^{\,3} D_2 = 1$ and $3X_1 + X_2 = 0$ are satisfied by D_1, D_2 and X_1, X_2, and

$$D_1 = d^{1/4}, D_2 = d^{-3/4}, \quad X_1 = \frac{1}{4}x, X_2 = -\frac{3}{4}x \quad .$$

From above relations, if $D_1 = D$, then $D_2 = D^{-3}$ and if $X_1 = X$, then $X_2 = -3X$.

Then equations (3.18) and (3.19) can be written as

$$a = D V^{1/4} \exp\left(X \int \frac{dt}{V} \right) \tag{3.20}$$

$$b = D^{-3} V^{1/4} \exp\left(-3X \int \frac{dt}{V} \right) \tag{3.21}$$

Now adding 3 times equation (3.4), (3.5) and 4 times equation (3.3) gives

$$3\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + 6\left(\frac{\dot{a}}{a}\right)^2 + 6\frac{\dot{a}\dot{b}}{ab} = \frac{32}{3}\pi(\rho - p) \tag{3.22}$$

From equation (3.7) we have

$$\frac{\ddot{V}}{V} = 3\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + 6\left(\frac{\dot{a}}{a}\right)^2 + 6\frac{\dot{a}\dot{b}}{ab}. \tag{3.23}$$

From equations (3.22), (3.23) and (3.13) we get

$$\frac{\ddot{V}}{V} = \frac{32}{3}\pi(1-\gamma)\rho, \quad \gamma \neq 1. \tag{3.24}$$

Substituting equation (3.15) in equation (3.24) we get

$$\frac{\ddot{V}}{V} = \frac{32\pi\alpha^2 B^2 (1-\gamma)}{3\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)} V^{2\left(\frac{\alpha^2}{f} - 1\right)}. \tag{3.25}$$

This further gives

$$V = \left\{ B\left(f - \alpha^2\right) \left[\frac{32\pi(1-\gamma)}{3\left(2\frac{\alpha^2}{f} - 1 + \gamma\right)} \right]^{1/2} \right\}^{\frac{f}{f-\alpha^2}} t^{\frac{f}{f-\alpha^2}}. \tag{3.26}$$

Substituting equation (3.26) in equation (3.12) we get

$$g = \frac{1}{\left(f - \alpha^2\right)}\left[\frac{32\pi(1-\gamma)}{3\left(2\alpha^2 - f + \gamma f\right)}\right]^{-1/2}\frac{1}{t}. \tag{3.27}$$

Also from equation $\dot{C}(V) = g(V)$, we get

$$C = \frac{1}{\left(f - \alpha^2\right)}\left[\frac{32\pi(1-\gamma)}{3\left(2\alpha^2 - f + \gamma f\right)}\right]^{-1/2}\log t \quad , \quad \gamma \neq 1. \tag{3.28}$$

Substituting equation (3.26) in equation (3.15) the homogenous mass density becomes

$$\rho = \frac{3\alpha^2 f}{32\pi(1-\gamma)\left(f - \alpha^2\right)^2}\frac{1}{t^2} \quad , \quad \gamma \neq 1. \tag{3.29}$$

Using equation (3.13), the pressure becomes

$$p = \frac{3\alpha^2 \gamma f}{32\pi(1-\gamma)\left(f - \alpha^2\right)^2}\frac{1}{t^2}, \quad \gamma \neq 1. \tag{3.30}$$

From equations (3.29) and (3.30), it is observed that

(i) when time $t \to \infty$, we get, density and pressure tending to zero i.e. the model reduces to vacuum.

(ii) when $f = \alpha^2$, there is singularity in density and pressure.

Using equation (3.25) in equations (3.18) and (3.19) we get,

$$a(t) = DK^{1/4} t^{\frac{f}{4(f-\alpha^2)}} \exp\left[\frac{X}{K}\left(1 - \frac{f}{\alpha^2}\right)t^{\left(\frac{\alpha^2}{\alpha^2 - f}\right)}\right], \tag{3.31}$$

$$b(t) = D^{-3}K^{1/4} t^{\frac{f}{4(f-\alpha^2)}} \exp\left[-\frac{3X}{K}\left(1 - \frac{f}{\alpha^2}\right)t^{\left(\frac{\alpha^2}{\alpha^2 - f}\right)}\right], \tag{3.32}$$

where $K = \left\{B\left(f - \alpha^2\right)\left[\frac{32\pi(1-\gamma)}{3\left(2\alpha^2 - f + \gamma f\right)}\right]^{1/2}\right\}^{\frac{f}{f-\alpha^2}}$.

Here D and X are constants.

IV. PHYSICAL PROPERTIES

We define $r = \left(a^3 b\right)^{\frac{1}{4}}$ as the average scale factor so that the Hubble's parameter in our anisotropic models may be defined as

$$H = \frac{\dot{r}}{r} = \frac{1}{4}\left(H_1 + H_2 + H_3 + H_4\right),$$

where H_i are directional Hubble's factors in the direction of x^i s respectively given by

$$H_1 = H_2 = H_3 = \frac{\dot{a}}{a}, H_4 = \frac{\dot{b}}{b}.$$

The expansion scalar θ is given by

$$\theta = 4H$$

$$\theta = \left(\frac{f}{f-\alpha^2}\right)\frac{1}{t}. \tag{4.1}$$

The mean anisotropy parameter is given by

$$A = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2$$

$$A = \frac{48X^2}{K^2}\left(\frac{f-\alpha^2}{f}\right)^2 \quad t^{2\left(\frac{\alpha^2}{\alpha^2 - f}\right)}. \tag{4.2}$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} H_i^2 - 4H^2\right) = \frac{4}{2}AH^2$$

$$\sigma^2 = \frac{6X^2}{K^2} t^{2\left(\frac{f}{\alpha^2 - f}\right)}. \tag{4.3}$$

The deceleration parameter q is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$$

$$q = 3 - \frac{4\alpha^2}{f}. \tag{4.4}$$

Here X is constant and $\Delta H_i = H_i - H$.

If $f > \alpha^2$ then for large t, the model tends to isotropic case.

Case I: $\gamma = 0$ ***(Dust Model)***

In this case, we obtain the values of various parameters as

$$g = \frac{1}{f-\alpha^2}\left[\frac{3\left(2\alpha^2 - f\right)}{32\pi}\right]^{1/2}\frac{1}{t}$$

.

$$C = \frac{1}{\left(f-\alpha^2\right)}\left[\frac{3\left(2\alpha^2-f\right)}{32\pi}\right]^{1/2}\log t$$

$$\rho = \frac{3\alpha^2 f}{32\pi\left(f-\alpha^2\right)^2}\frac{1}{t^2}$$

$$a(t) = DK_1^{1/4}t^{\frac{f}{4(f-\alpha^2)}}\exp\left[\frac{X}{K_1}\left(1-\frac{f}{\alpha^2}\right)t^{\left(\frac{\alpha^2}{\alpha^2-f}\right)}\right],$$

$$b(t) = D^{-2}K_1^{1/4}t^{\frac{f}{4\left(f-\alpha^2\right)}}\exp\left[-\frac{3X}{K_1}\left(1-\frac{f}{\alpha^2}\right)t^{\left(\frac{\alpha^2}{\alpha^2-f}\right)}\right],$$

where $K_1 = \left\{B\ \left(f-\alpha^2\right)\left[\frac{32\pi}{3\left(2\alpha^2-f\right)}\right]^{1/2}\right\}^{\frac{f}{f-\alpha^2}}$

Here D and X are constants.

In this case, the expansion scalar θ is given by

$$\theta = \left(\frac{f}{f-\alpha^2}\right)\frac{1}{t}.$$

The mean anisotropy parameter is given by

$$A = \frac{48X^2}{K_1^2}\left(\frac{f-\alpha^2}{f}\right)^2 \quad t^{2\left(\frac{\alpha^2}{\alpha^2-f}\right)}.$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{6X^2}{K_1^2}t^{2\left(\frac{f}{\alpha^2-f}\right)}.$$

The deceleration parameter q is given by

$$q = 3 - \frac{4\alpha^2}{f}$$

If $f > \alpha^2$, this model tends to isotropy for large t.

For $f = \alpha^2$,we get ,singularity in density.

For $3f = \alpha^2$, we get , $\lim_{t\to\infty}\frac{\sigma}{\theta} \neq 0$. Hence anisotropy is maintained throughout.

For $3f > \alpha^2$, we get , $\lim_{t\to\infty}\frac{\sigma}{\theta} \to 0$. Hence model isotropizes for large value of t .

In this case, for $3f = 4\alpha^2$, we get , $q = 0$ indicating that the universe is decelerating.

Also, for $3f < 4\alpha^2$, we get , $q < 0$ indicating that the universe is accelerating which is consistent with present day observation.

Case II : $\gamma = \frac{1}{3}$ (Disordered Radiation Model)

In this case, we obtain the values of various parameters as

$$g = \frac{1}{f - \alpha^2}\left[\frac{3\left(3\alpha^2 - f\right)}{32\pi}\right]^{1/2}\frac{1}{t}$$

$$C = \frac{1}{f - \alpha^2}\left[\frac{3\left(3\alpha^2 - f\right)}{32\pi}\right]^{1/2}\log t$$

$$\rho = \frac{9\alpha^2 f}{64\pi\left(f - \alpha^2\right)^2}\frac{1}{t^2},$$

$$a(t) = DK_2^{1/4} t^{\frac{f}{4\left(f - \alpha^2\right)}} exp\left[\frac{X}{K}\left(1 - \frac{f}{\alpha^2}\right)t^{\left(\frac{\alpha^2}{\alpha^2 - f}\right)}\right]$$

$$b(t) = D^{-3} K_2^{1/4} t^{\frac{f}{4\left(f - \alpha^2\right)}} exp\left[-\frac{3X}{K}\left(1 - \frac{f}{\alpha^2}\right)t^{\left(\frac{\alpha^2}{\alpha^2 - f}\right)}\right]$$

where $K_2 = \left\{B\left(f - \alpha^2\right)\left[\frac{32\pi}{3\left(2\alpha^2 - f\right)}\right]^{1/2}\right\}^{\frac{f}{f - \alpha^2}}$

Here D and X are constants.

In this case, the expansion scalar θ is given by

$$\theta = \left(\frac{f}{f - \alpha^2}\right)\frac{1}{t}.$$

The mean anisotropy parameter is given by

$$A = \frac{48X^2}{K_2^2}\left(\frac{f - \alpha^2}{f}\right)^2 \quad t^{2\left(\frac{\alpha^2}{\alpha^2 - f}\right)}.$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{9X^2}{K_2^2} t^{2\left(\frac{f}{\alpha^2 - f}\right)}.$$

The deceleration parameter q is given by

$$q = 3 - \frac{4\alpha^2}{f}$$

For $f = \alpha^2$,we get ,singularity in density.

For $3f = \alpha^2$, we get, $\lim_{t\to\infty} \frac{\sigma}{\theta} \neq 0$. Hence anisotropy is maintained throughout.

For $3f > \alpha^2$, we get, $\lim_{t\to\infty} \frac{\sigma}{\theta} \to 0$. Hence model isotropizes for large value of *t*.

Here in this case, for $3f = 4\alpha^2$, we get , $q = 0$ indicating that the universe is decelerating.

Also, for $3f < 4\alpha^2$, we get , $q < 0$ indicating that the universe is accelerating which is consistent with present day observation.

In all cases, One should note that Creation field *C* is proportional to time *t* . That is, the creation of matter increases as time increases.

V. CONCLUSIONS

Hoyle and Narlikar's *C*-field cosmology is studied and extended to the framework of Kaluza-Klein higher dimensional space time. This study is particularly interesting in the sense that we have here presented a case where the cosmology manifestly has a Big Bang type of singularity. It emphasizes the fact that mere inclusion of *C*-field is no guarantee against the occurrence of singular epoch. Further, we have shown that in line with the physical requirement our model admits a solution with a decelerating phase in the early era followed by a later accelerated expansion. This is in conformity with the present day observational status.

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