PLANE SYMMETRIC DOMAIN WALLS AND COSMIC STRINGS IN SCALE-COVARIANT THEORY OF GRAVITATION

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Plane symmetric thick domain walls and cosmic strings are considered in Canuto *et al* (1977) formulated scale-covariant theory of gravitation. It is shown that, in this theory, thick domain walls and cosmic strings do not exist.

Key words: cosmic strings, domain walls, scale-covariant theory.

1. INTRODUCTION

In recent years there has been lot of interest in several alternative theories of gravitation. The most important among then are scalar-tensor theories of gravitation formulated by Brans-Dicke (1961), Nordt-Vedt (1970), Ross (1972), Rosen (1973), Dunn (1974) and Saez-Ballester (1985). All versions of the scalar-tensor theories are based on the introduction of a scalar field ϕ into the formulation of general relativity. This scalar field together with the metric tensor field then forms a scalar-tensor field representing the gravitational field.

Canuto *et al.* (1977) formulated a scale-covariant theory of gravitation which also admits a variable G and which is a viable alternative to general relativity. In the scale-covariant theory, Einstein's field equations are valid in gravitational units where as physical quantities are measured in atomic units. The metric tensors in the two systems of units are related by a conformal transformation.

$$\overline{g}_{ij} = \phi^2\left(x^k\right)g_{ij},\tag{1}$$

wherein Latin indices takes values 1, 2, 3 and 4, bars denote gravitational units and unbar denotes atomic quantities. The gauge function ϕ $(0 < \phi < \infty)$ in its most general formulation is a function of all space time coordinates. Thus, using the conformal transformation of the type given by (1), Canuto *et al.* (1977) transformed the usual Einstein equation into:

$$R_{ij} - \frac{1}{2}Rg_{ij} + f_{ij}\left(\phi\right) = -8\Pi G\left(\phi\right)T_{ij} + \Lambda\left(\phi\right)g_{ij}\tag{2}$$

where

$$\phi^2 f_{ij} = 2\phi\, \phi_{i;j} - 4\phi_i\, \phi_j - g_{ij}\left(\phi\phi_{;k}^k - \phi^k\phi_k\right),\qquad(3)$$

where R_{ij} is the Ricci tensor, R the Ricci scalar, Λ the cosmological constant, G the gravitational 'constant' and T_{ij} is the energy momentum tensor. A semicolon denotes covariant derivative and ϕ_i denotes ordinary derivative with respect to x^i. A particular feature of this theory is that no independent equation for ϕ exists. The possibilities that have been considered for gauge functions ϕ are

$$\phi(t) = \left(\frac{t_o}{t}\right)^{\in}, \qquad \in = \pm 1, \pm \frac{1}{2}\ ,\qquad(4)$$

where t_0 is constant. The form $\phi \sim t^{\frac{1}{2}}$ is the one most favored to fit observations.

The energy conservation equation for perfect fluid is given by

$$\rho_4 + (\rho + p)u_{;i}^i = -\rho\frac{(G\phi)_4}{G\phi} - 3p\frac{\phi_4}{\phi}.\qquad(5)$$

A detailed discussion of scale covariant theory is contained in the work of Canuto *et al.* (1977b), Beeshan (1986, a, b, c), Reddy and Venkateswarlu (1987), Reddy *et al.* (2002). Reddy and Venkateswarlu (2004) have investigated several aspects of this theory of gravitation with the perfect fluid matter distribution as source.

The study of cosmic strings and domain walls has received considerable attention in cosmology since it plays an important role in structure formation and evolution of the universe. Cosmic strings and domain walls are topological defects associated with spontaneous symmetry breaking whose plausible production site is cosmological phase transitions in the early universe. The gravitational effects of cosmic strings have been extensively discussed by Vilenkin (1981), Gott (1985), Leteliar (1983), Satchel (1980) and Adhav *et al.* (2007a, b) in general relativity. Also, Tikekar *et al.* (1994) have presented a class of cylindrically symmetric molds in string cosmology.

In particular, the domain walls have become important in recent years from cosmological stand-point when a new scenario of galaxy formation has been proposed by Hillet *et al.* (1989). Vilenkin (1983), Ispeu and Sikivie (1984), Widrow (1989), Goets (1990), Mukherjee (1993), Wang (1994), Rahaman *et al.* (2001), Reddy and Subbarao (2006) and Adhav (2007c) are some of the authors who have investigated several aspects of domain walls.

The purpose of the present work is to study plane symmetric cosmological models in a scale covariant theory of gravitation with cosmic strings and domain walls. Our paper is organized as follows. In Section 2, we have discussed plane symmetric string cosmological models in the scale-covariant theory of

gravitation. Section 3 contains discussion on the thick domain walls in plane symmetric space time. The last section contains some conclusions.

2. COSMIC STRINGS

We consider the energy momentum tensor for cosmic string source as:

$$T_j^i = \rho\, u^i u_j - \lambda x^i x_j, \tag{6}$$

where ρ is the rest energy density of the cloud of strings with massive particles attached to them, $\rho = \rho_p + \lambda$, ρ_p being the rest energy of the particles attached to the strings and λ the tension density of the system of strings. u^i describes the cloud four velocity and x^i represents the direction of strings.

We consider the plane symmetric metric

$$ds^2 = dt^2 - A^2\left[dx^2 + dy^2\right] - B^2 dz^2, \tag{7}$$

where A & B are functions of 't' only. Orthonormalism of u^i and x^i is given as

$$u^i u_{i=1}, x^i x_j = 0, \qquad x^i x_j = -1. \tag{8}$$

In the co-moving coordinate system, we have from (6)

$$T_1^1 = T_2^2 = 0, \quad T_3^3 = \lambda, \quad T_4^4 = \rho \quad \text{and} \quad T_j^i = 0 \quad \text{for } i \neq j. \tag{9}$$

The quantities ρ and λ depend on t only. Here the string source is along z-axis which is the axis of symmetry.

Now, with help of equations (6), (8) and (9), the field equations (2), (3) and (5) for the metric (7), with zero cosmological constant can be written as

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{B_4 \phi_4}{B\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 0 \tag{10}$$

$$2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 + 2\frac{A_4 \phi_4}{A\phi} - \frac{B_4 \phi_4}{B\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\lambda \tag{11}$$

$$2\frac{A_4 B_4}{AB} + \left(\frac{A_4}{A}\right)^2 + 2\frac{A_4 \phi_4}{A\phi} + \frac{B_4 \phi_4}{B\phi} - \frac{\phi_{44}}{\phi} + 3\left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\rho \tag{12}$$

$$\rho_4 + (\rho + \lambda)\left(2\frac{A_4}{A} + \frac{B_4}{B}\right) = -\rho\frac{G_4}{G} - (\rho + 3\lambda)\frac{\phi_4}{\phi}, \tag{13}$$

where the suffix 4 after an unknown function denotes differentiation with respect to 't'.

The field equations (10)–(12) are three equations in six unknowns A, B, ϕ, ρ, λ, $G(\phi)$. Hence, to get a determinate solution we have to assume a relation between metric potentials A and B as

$$A = \alpha\beta, \quad \text{where } \alpha = \text{constant} \tag{14}$$

Using equation (14), the set of equations (10)–(13) reduces to

$$2\frac{A_{44}}{AB} + \left(\frac{A_4}{A}\right)^2 + \frac{A_4\phi_4}{A\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 0 \tag{15}$$

$$2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 + \frac{A_4\phi_4}{A\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\lambda \tag{16}$$

$$2\left(\frac{A_4}{A}\right)^2 + 3\frac{A_4\phi_4}{A\phi} - \frac{\phi_{44}}{\phi} + 3\left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\rho \tag{17}$$

$$\rho_4 + 3(\rho + \lambda)\frac{A_4}{A} = -\rho\frac{G_4}{G} - (\rho + 3\lambda)\frac{\phi_4}{\phi} \tag{18}$$

From equations (15) and (16),

we get $\quad\quad\quad\quad\quad\quad\quad\quad \lambda = 0.$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ (19)

In the literature [Leteliar (1983)], we have the equations of state for string models as

$$\rho = \lambda \quad \text{(geometric or Nambu string)} \tag{20}$$

$$\rho = (1 + \omega)\lambda \quad \text{(ρ – string or Takabayaski string)} \tag{21}$$

$$\rho + \lambda = 0 \text{ (Reddy string)} \tag{22}$$

From equations (19), (20), (21) and (22), we get, $\rho = 0$, which shows that in scale covariant theory neither geometric strings nor *p* – strings nor Reddy strings survive. Hence, we observe that the geometric strings, *p* – strings and Reddy strings do not exists in the scale-covariant theory of gravitation.

3. THICK DOMAIN WALLS

A thick domain wall can be viewed as a soliton like solution of the scalar field equations coupled with gravity. There are two ways of studying thick domain walls. One way is to solve gravitational field equations with an energy momentum tensor describing a scalar field ψ with self-interactions contained in a potential $v(\psi)$ given by

$$\psi_i \psi_j - g_{ij} \left[\frac{1}{2} \psi_k \psi^k - v(\psi) \right] \tag{23}$$

Second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho(g_{ij} + \omega_i \omega_j) + p\omega_i \omega_j, \qquad \omega_i \omega_j = -1, \tag{24}$$

where ρ is the energy density of the walls, p is the pressure in the direction normal to the plane of the wall and ω_i is a unit space-like vector in the same direction.

Here we use the second approach to study the thick domain walls in scale-covariant theory of gravitation.

In co-moving coordinate system we have from equation (24)

$$T_1^1 = T_2^2 = T_4^4 = \rho, \quad T_3^3 = -p \quad \text{and} \quad T_j^i = 0 \quad \text{for} \quad i \ne j. \tag{25}$$

Here pressure is taken in the direction of z-axis. The quantities ρ and p depends on t only.

Now, the field equations (2), (3) and (5) [with zero cosmological constant] for metric (7), with the help of equations (24) and (25) can be written as

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{B_4 \phi_4}{B\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G \rho \tag{26}$$

$$2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 + 2\frac{A_4 \phi_4}{A\phi} - \frac{B_4 \phi_4}{B\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = -8\pi G p \tag{27}$$

$$2\frac{A_4 B_4}{AB} + \left(\frac{A_4}{A}\right)^2 + 2\frac{A_4 \phi_4}{A\phi} + \frac{B_4 \phi_4}{B\phi} - \frac{\phi_{44}}{\phi} + 3\left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G \rho \tag{28}$$

$$\rho_4 + (\rho + p)\left(2\frac{A_4}{A} + \frac{B_4}{B}\right) = -\rho\frac{G_4}{G} - (\rho + 3p)\frac{\phi_4}{\phi} \tag{29}$$

The field equations (26)–(29) are four equations in unknowns A, B, ϕ, ρ, λ and $G(\phi)$.

Hence to obtain a determinate solution, we assume a relation (14) between metric potentials and we also assume the equation of state

$$\rho = p \tag{30}$$

Using equations (14) and (30), the set of field equations (26)–(29) reduces to

$$2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 + \frac{A_4 \phi_4}{A\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G \rho \tag{31}$$

$$2\frac{A_{44}}{A}+\left(\frac{A_4}{A}\right)^2+\frac{A_4\phi_4}{A\phi}+\frac{\phi_{44}}{\phi}-\left(\frac{\phi_4}{\phi}\right)^2=-8\pi G\rho \tag{32}$$

$$3\left(\frac{A_4}{A}\right)^2+3\frac{A_4\phi_4}{A\phi}-\frac{\phi_{44}}{\phi}+3\left(\frac{\phi_4}{\phi}\right)^2=8\pi G\rho \tag{33}$$

$$\rho_4+3(\rho+p)\frac{A_4}{A}=-\rho\frac{G_4}{G}-(\rho+3p)\frac{\phi_4}{\phi}. \tag{34}$$

With the help of equations (31) and (32)

we get
$$\rho=0. \tag{35}$$

By using equation of state $\rho = p$,

we get,
$$\rho = 0 = p \tag{36}$$

which shows that, stiff or self gravitating domain walls do not survive in scale covariant theory of gravitation.

CONCLUSION

We have shown that plane symmetric cosmic strings models which represent Nambu strings (geometric strings), p-strings and Reddy strings do not survive in the scale covariant theory of gravitation formulated by Canuto *et al.* (1977) when we assume a relation between metric coefficients. We have also shown that, in this particular case, the self-gravitating or stiff domain walls do not exists.

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