IJFEAT

INTERNATIONAL JOURNAL FOR ENGINEERING APPLICATIONS AND TECHNOLOGY

Bianchi type-III Domain Walls with Electromagnetic field in General Relativity

M. R. Ugale[1], A. S. Nimkar[2], A. M. Pund[3]
[1]Department of Mathematics, Sipna's College of Engineering,Amravati (india).
mohini_261@redifmail.com
[2]Department of Mathematics, Shri Dr.R.G.Rathod Arts and Science College, Murtizapur (India).
anilnimkar@gmail.com
[3]Department of Mathematics, Shivaji Science College, Congress Nagar, Nagpur 440012(India).
ashokpund64@rediffmail.com

Abstract: In this paper. we have examined domain walls with electromagnetic field in Bianchi type-III space time . Exact cosmological model is obtained. Also, we have discussed the features of the obtained solutions.

Key words : *Bianchi type, Electromagnetic field, Relativity*

1]Introduction:

Bianchi type cosmological model are important in the sense that these are homogenous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universe have a greater generally than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogenous and anisotropic cosmologies.

The domain walls have become important in recent years from cosmological stand point when a new scenario of galaxy formation has been proposed by Hillet *et al* (1989). According to them the formation of galaxies are due to domain walls produced during phase transitions after the time of recombination of matter and radiation. So far a considerable amount of work has been done on domain walls. Widrow (1989), Goets (1990), Mukherjee (1993), Wang (1994), Rahaman et al (2001), Rahaman (2002), Reddy & Subbarao (2006) are some of the authors who have investigated several aspects of domain walls. Adhav *et al*.(2007) have investigated several aspects of strings and domain walls for Bianchi type space times. Recently, Pund *et.al.*(2014) have studied Cosmological model with Domain walls including normal Matter.

The purpose of the present work is to obtain Bianchi type-III cosmological model in presence of domain walls with electromagnetic field. Our paper is organized as follows. In section 2, we derive the Einstein's field equations and their solutions. section 3, is mainly concerned with the physical properties of the model. The last section contains some conclusion.

2]Einstein's field equations and their solutions for the domain walls with Electric field:

Let's consider the Bianchi type-III space-time in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2ax}dy^2 - C^2dz^2 \tag{1}$$

Where A, B ,C are functions of time t alone and a is constant.

The total energy-momentum tensor T_{ij} is assumed to be the sum of two parts, ${}^{D}T_{ij}$ and ${}^{E}T_{ij}$, for domain wall and electromagnetic contributions respectively,

$$T_{ij} = {}^{D}T_{ij} + {}^{E}T_{ij} \tag{2}$$

The energy momentum tensor of the domain wall is given by

$${}^{D}T_{ij} = (\rho + p)u_i u_j - pg_{ij} \quad , \tag{3}$$

Where u^i is the flow vector satisfying $g_{ijj}u^i u^j = 1$

 ISSN: 2321-8134

The energy momentum tensor of the electromagnetic field

$$ {}^{E}T_{ij} = \frac{1}{4\pi}\left[-F_{is}F_{jp}g^{sp} + \frac{1}{4}g_{ij}F_{sp}F^{sp} \right] \tag{4} $$

Where F_{ij} is the Electromagnetic field tensor defined by the four potential A_i as

$$ F_{ij} = A_{j,i} - A_{i,j} \tag{5} $$

Here assume that F_{12} is the only non-vanishing components of F_{ij}

$$ \frac{\partial}{\partial x^{j}}\left(F^{ij}\sqrt{-g}\right) = 0 \tag{6} $$

Leads to

$$ \frac{\partial}{\partial y}\left(F^{12}ABCe^{-ax}\right) = 0 $$

Thus $F_{12} = He^{-ax}$ where *H* and *a* are constant.

The Einstein field equation can be expressed as

$$ R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} \tag{7} $$

Using the line element (1), from equations (2)-(6), we get

$$ \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = -8\pi p - \frac{H^2}{A^2B^2} \tag{8} $$

$$ \frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} = -8\pi p - \frac{H^2}{A^2B^2} \tag{9} $$

$$ \frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{a^2}{A^2} = -8\pi p + \frac{H^2}{A^2B^2} \tag{10} $$

$$ \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} - \frac{a^2}{A^2} = 8\pi p + \frac{H^2}{A^2B^2} \tag{11} $$

$$ \frac{A_4}{A} - \frac{B_4}{B} = 0 \tag{12} $$

Where the subscript '4' after *A*,*B* and *C* denotes ordinary differentiation with respect to *t*.

From equation (12), we have

$A = \mu B$, where μ is a constant of integration (13)

With the help of equation (13), the set of equation (8)-(11) reduces to

 ISSN: 2321-8134

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}=-8\pi p-\frac{H^2}{\mu^2B^4} \tag{14}$$

$$2\frac{B_{44}}{B}+\left(\frac{B_4}{B}\right)^2-\frac{a^2}{\mu^2B^2}=-8\pi p+\frac{H^2}{\mu^2B^4} \tag{15}$$

$$\left(\frac{B_4}{B}\right)^2+2\frac{B_4C_4}{BC}-\frac{a^2}{\mu^2B^2}=8\pi\rho+\frac{H^2}{\mu^2B^4} \tag{16}$$

From equation (14)-(16). We get

$$A=\mu\left[K_3{}^2T^2-\beta^2K_4{}^2\right]^{\frac{1}{2}} \tag{17}$$

$$B=\mu\left[K_3{}^2T^2-\beta^2K_4{}^2\right]^{\frac{1}{2}} \tag{18}$$

$$C=\mu\left[K_3{}^2T^2-\beta^2K_4{}^2\right]^{\frac{1}{2n}} \tag{19}$$

Where $T=t+K_2$, $K_3{}^2=\frac{n^2\quad a^2}{(n^2-1)\mu^2}$, $K_4{}^2=\frac{n^2}{K_3{}^2}$

$$p=-\frac{1}{16\pi}\left[\frac{\left(3k_3{}^2+\frac{k_3{}^2}{n}-\frac{a^2}{\mu^2}\right)}{\left(k_3{}^2T^2-\beta^2K_4{}^2\right)}+\left(\frac{1}{n^2}+\frac{1}{n}-2\right)\frac{T^2K_3{}^4}{\left(K_3{}^2T^2-\beta^2K_4{}^2\right)^2}\right] \tag{20}$$

$$\rho=\frac{1}{8\pi}\left[\frac{K_3{}^4T^2}{\left(k_3{}^2T^2-\beta^2\mu_4{}^2\right)}\left(1+\frac{2}{n}+\frac{1}{2n^2}+\frac{1}{2n}\right)-\frac{1}{\left(K_3{}^2T^2-\beta^2K_4{}^2\right)}\left[\left(\frac{3}{2}+\frac{1}{2n}-1-\frac{1}{n}\right)k_3{}^2+\frac{1}{2}\frac{a}{\mu^2}\right]\right] \tag{21}$$

$$H^2=\mu^2\left(K_3{}^2T-\beta^2K_4{}^2\right)\left[\left(\frac{3}{2}+\frac{1}{2n}-1-\frac{1}{n}\right)K_3{}^2-\frac{a^2}{2\mu^2}\right]-\mu^2T^2K_3{}^4\left(\frac{1}{2n^2}+\frac{1}{2n}\right) \tag{22}$$

With the help of (17-19) the Bianchi type-III Domain Walls with Electromagnetic field model can be expressed as

$$ds^2=dt^2-\mu^2\left(K_3^2T^2-B^2K_4^2\right)\left(dx^2-e^{-2ax}dy^2\right)-\mu^2\left(K_3^2T^2+\beta^2K_4^2\right)^{\frac{1}{a}}dz^2 \tag{23}$$

3]Physical Properties of the model:

The physical quantities that are important in cosmology are spatial volume v^3, the expansion scalar θ, shear scalar σ^2 Which have the following expressions for the model (23) as given below:

spatial volume : $v3 = \sqrt{-g} = \mu\left[K_3^2T^2 - \beta^2 K_4^2\right]^{\left(1+\frac{1}{2n}\right)} e^{-ax}$ (24)

scalar expansion : $\theta = U^i_{;i} = \left(\frac{2n+1}{3n}\right)\frac{TK_3^2}{K_3^2T^2 - \beta^2 K_4^2}$ (25)

shear scalar $\sigma^2 = \frac{1}{6}\theta^2 = \frac{1}{6}\left(\frac{2n+1}{3n}\right)^2 \frac{T^2K_3^4}{\left(K_3^2T^2 - \beta^2 K_4^2\right)^2}$ (26)

It may be observed here that at initial moment (T = 0), the spatial volume will be zero. The expansion scalar θ and shear scalar σ^2 tends to infinity as $T \rightarrow 0$. Whereas when $T \rightarrow \infty$, the spatial volume becomes infinitely large but expansion scalar and shear scalar tends to zero.

4] Conclusion:

It is interesting to note that as T gradually increases, the scalar expansion θ and shear scalar σ^2 decrease and finally they vanish when $T \rightarrow \infty$. This fact implies that our solution represents the early stages of evolution of the universe which is analogous. Since lim $T \rightarrow \infty, \frac{\sigma^2}{\theta^2} \neq 0$ being independent of cosmic time. That implies the model does not approach isotropy for large values of T. The model is expanding shearing, non-rotating and has no initial singularities.

References:

1.Hillet, C.T., Schram, D.N. and Fra, J.N.: Nucl. Part. Phys., **19,** 25 (1989). Widrow, L.M. : Phys. Rev. **D.39**, 3571 (1989).

2.Widrow, L.M. : Phys. Rev. **D.39**, 3571 (1989).

3.Goetz, G. : J.Math. Phys **31**, 2683 (1990).

 ISSN: 2321-8134

4.Mukherji, M. : Class. Quan. Grav. **10**,131 (1993).

5.Wang, A. : Mod. Phys. Lett. **39**,3605 (1994).

6.Rahaman, F., Chakraborty, S., Kalam, M. : Int. J.Mod. Phys. D **10**, 735 (2001).

7.Rahaman F. : Astrophys. Space Sci. **282**, 625 (2002).

8.Reddy, D.R.K. Rao, M.V.S. : Astrophys. Space Sci. **302**, 157 (2006).

9.Adhav K.S., Nimkar A.S.,Naidu R.L.: Astrophys space sci.312,165-169(2007).

10. Pund, A. M., Avachar,G. R.: Prespace time journal Vol.5,3,207-211(2014).