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WET DARK FLUID COSMOLOGICAL MODEL IN SAEZ-BALLESTER THEORY

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Abstract
Axially symmetric Bianchi Type-I wet dark fluid cosmological model is obtained in scalar tensor theory of gravitation proposed by Saez and Ballester [1]. Some physical and kinematical properties of the model are discussed.

Introduction
Bianchi type cosmological model are important in the sense that these are homogenous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universe have a greater generally than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogenous and anisotropic cosmologies

Einstein's general theory of relativity has been successful in describing gravitational phenomena. It has also served as a basis for models of the universe. However, since Einstein first published his theory of gravitation, there have been many criticisms of general relativity because of the lack of certain 'desirable' features in the theory. For example Einstein himself pointed out that general relativity does not account satisfactory for inertial properties of matter, i.e. Mach's principle is not substantiated by general relativity. So in recent years there has been lot of interest in several alternative theories of gravitation. The most important among them are scalar-tensor theories of gravitation formulated by Brans and Dicke [2] , Nordvedt [3] and Saez –Ballester [1].

All version of the scalar-tensor theories are based on the introduction of a scalar field ϕ into the formulation of general relativity, this scalar field together with metric tensor field then forms a scalar-tensor field representing the gravitational field.

The field equations given by Saez and Ballester [1] for the combined Scalar and tensor fields are

$$G_{ij} - \omega\phi^{n}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \quad (1)$$

$$2\phi^{n}\phi^{i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \quad (2)$$

Where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor, R_{ij} is the Ricci tensor, R is the scalar curvature, n an arbitrary constant , ω is a dimensionless coupling constant and T_{ij} is the matter energy-momentum tensor.
The equation of motion

$$T^{ij}_{;j} = 0 \quad , \quad (3)$$

is a consequence of field equation (1) and (2).
R. Holman and Siddartha Naidu [4] studied wet dark fluid (WDF) as a model for dark energy. This model was in the spirit of the generalized chaplygin gas (GCG), where a physically motivated equation of state was offered with properties relevant for the dark energy problem. Here the motivation stems from an empirical equation of state proposed by Tait [5] and Hayword [6] to treat water and aqueous solution. The equation of state for WDF is very simple.

$$p_{WDF} = \gamma\left(\rho_{WDF} - \rho^{*}_{WDF}\right), \qquad (4)$$

and is motivated by the fact that it is a good approximation for many fluids, including water, in which the internal attraction of the molecules makes negative pressures possible. One of the virtues of this model is that the square of the sound speed, C_S^2 which depends on $\frac{\partial p}{\partial \rho}$, can be positive, even while giving rise to cosmic acceleration in the current epoch. In real fluid negative pressures eventually lead to a breakdown of equation (1) as a Phemenological equation. We will show that this model can be made consistent with the most recent SNIa data, the WMAP results as well as the constraints coming from measurements of the matter power spectrum. The parameters γ and ρ^* are taken to be positive and we restrict ourselves to $0 \le \gamma \le 1$. Note that it C_s denotes the adiabatic sound speed in WDF, then $\gamma = C_s^2$.

To find the WDF energy density, we use the energy conservation equation

$$\dot{\rho}_{WDF} + 3H\left(p_{WDF} + \rho_{WDF}\right) = 0 \qquad (5)$$

$$\Rightarrow \qquad \rho_{WDF} = \frac{\gamma}{1+\gamma}\rho^{*} + \frac{D}{V^{(1+\gamma)}},$$

where *D* is the constant of integration and *V* is the volume expansion. WDF naturally includes two components, a piece that behaves as a cosmological constant as well as a piece that red shifts as a standard fluid with an equation of state $p = \gamma\rho$. We can show that it we take $D > 0$, this fluid will never violate the strong energy condition

$p + \rho \ge 0$. Thus, we get

$$p_{WDF} + \rho_{WDF} = (1+\gamma)\rho_{WDF} - \gamma\rho^{*}$$

$$= D(1+\gamma)\frac{D}{V^{(1+\gamma)}} \ge 0 .$$

A detailed discussion of Saez - Ballester cosmological model is contained in the work of Singh and Agrawal [7], Shri Ram and Tiwari [8],Reddy and Venkateswara Rao[9], D.R.K.Reddy,C.S.V.V.R. Murthy, R.Venkateswarlu[10], Adhav*et al*[11,12], Ugale [13]and Pund & Nimkar [14].

Also, Adhav *et al* [15] have been studied in detailed for Einstein-Rosen universe with wet dark fluid, Chirde *et al*[16] studied Bianchi Type-I universe with wet dark fluid in scalar tensor theory of gravitation, Nimkar *et al*[17,18] have been discussed wet dark fluid in Lyra geometry and Ruban's background ,Mete *et al*[19] studied inhomogeneous wet dark fluid cosmological model and very recently Tade *et al* [20] discussed wet dark fluid cosmological model in bimetric theory of gravitation.

In this paper, we have obtained axially symmetric Bianchi type-I cosmological model in scalar tensor of gravitation proposed by Saez and Ballester in presence of a wet dark fluid. Some physical and kinematical properties of the cosmological model are also discussed.

Metric and field equations

We consider axially symmetric Bianchi Type-I metric

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2), \qquad (6)$$

where *A* and *B* are the functions of cosmic time *'t'* only.
The energy-momentum tensor for wet dark fluid is

$$T_{ij} = (\rho_{WDF} + p_{WDF})u_i u_j - p_{WDF} \qquad (7)$$

Where, u^i is flow vector satisfying . $g_{ij}u.^i u_j$.=1

The field equations (1-2) for the metric (6) with matter distribution (7) yield

$$2\frac{B_{44}}{B}+\left(\frac{B_4}{B}\right)^2-\frac{\omega}{2}\phi^n{\phi_4}^2=p_{WDF} \tag{8}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{\omega}{2}\phi^n{\phi_4}^2=p_{WDF} \tag{9}$$

$$2\frac{A_4B_4}{AB}+\left(\frac{B_4}{B}\right)^2+\frac{\omega}{2}\phi^n{\phi_4}^2=-\rho_{WDF} \tag{10}$$

$$\frac{A_4}{A}+2\frac{B_4}{B}+\frac{\phi_{44}}{\phi_4}+\frac{n}{2}\frac{\phi_4}{\phi}=0 \tag{11}$$

$$(\rho_{WDF})_4+\rho_{WDF}\left(\frac{A_4}{A}+2\frac{B_4}{B}\right)=0 \tag{12}$$

Solutions of the model

The set of field equation (8) to (12) are highly nonlinear in matter, in general , it is difficult to obtain the explicit solution of the field equation. The relation between metric coefficient *A* and *B.*

i.e. $A=B^n$, where *'n'* is constant (13) and

$$p_{WDF}=\rho_{WDF} \tag{14}$$

Using equation (13) and (14) an exact solution of the field equation are

$$A=k_4(k_2t+k_3)^{\frac{n}{k_1+1}},\qquad B=k_3(k_2t+k_3)^{\frac{1}{k_1+1}}, \tag{15}$$

where, $k_4=k_3^n$ and $k_3=(k_1+1)^{\frac{1}{k_1+1}}$.

$$p_{WDF}=\rho_{WDF}=\frac{k_5}{(k_3)^{n+2}(k_2t+k_3)^{\frac{n+2}{k_1+2}}} \tag{16}$$

The scalar field ϕ is given by

$$\phi=\left(\frac{n+2}{2}\right)^{\frac{2}{n+2}}\left[\left(\frac{k_1+1}{k_1-n-1}\right)k_7(k_2t+k_3)^{\frac{k_1-n-1}{k_1+1}}+k_8\right]^{\frac{2}{n+2}} \tag{17}$$

Through a proper choice of co-ordinates and constants the axially symmetric Bianchi Type-I Saez-Ballester wet dark model can be written as

$$ds^2=-\frac{dT^2}{{k_2}^2}+{k_4}^2(T)^{\frac{2n}{k_1+1}}dx^2+{k_3}^2(T)^{\frac{2}{k_1+1}}(dy^2+dz^2), \tag{18}$$

Where $T=(k_2t+k_3)$

Physical and kinematical Properties

The physical and kinematical quantities for the model (18) have the following expression

Proper volume $V^3 = \sqrt{-g} = k_9 (T)^{\frac{n+2}{k_1+1}}$ (19)

Expansion Scalar $(\theta) = \frac{k_{10}}{T}$, $k_{10} = \frac{(n+2)k_2}{3(k_1+1)}$ (20)

Shear scalar $(\sigma^2) = \frac{7k_{11}}{18T^2}$, $k_{11} = \left[\frac{(n+2)k_2}{3(k_1+1)}\right]^2$ (21)

The model (18) has no initial singularity, while the energy density and pressure given by (16) possess initial singularities. However, as T increases these singularities vanish. The proper volume of the model given by (19) shows the anisotropic expansion of the universe (18) with time. For the model (18), the expansion scalar θ and shear scalar σ^2 tends to zero as $T \to \infty$..

Conclusion

In this paper, we have considered axially symmetric Bianchi Type-I cosmological model in Saez-Ballester scalar tensor theory of gravitation in the presence of wet dark fluid. For solving the field equations we have assumed the equation $p_{WDF} = \rho_{WDF}$. The model (18) is free from singularities and it is expanding and decelerates in the standard way.

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