Article

Cosmological Model with Domain Walls Including Normal Matter

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Abstract

In this paper, we have examined domain walls include normal matter described by ρ_m and p_m as well as a domain tension σ in Kantowski-Sach space-time. Exact cosmological model is presented with the help of equation of state. Also, some physical and kinematical properties of the model are discussed.

Key words: Kantowski-Sach, spacetime, domain walls, normal matter.

1. Introduction

Kantowski-Sachs spacetime can be defined locally as those admitting a G_3 isometry group acting on two-dimensional space-like orbits of positive curvature and possessing an additional space-like Killing field which does not lie in those orbits. The Kantowski-Sachs spacetime is of some importance as they include relatively simple examples of spatially homogenous and spherically symmetrical space times that are anisotropic. A qualitative study of Kantowski-Sachs cosmological models, [Kantowski (1969); Kantowski and Sachs (1966)] has recently been performed by Weber (1984, 1985). Gron (1986) found an exact solution of the Einstein vacuum field equations with a cosmological constant. These models are spatially homogenous, have shear, and have no rotation.

The Kantowski-Sachs cosmological models containing a perfect fluid with a zero cosmological constant was analyzed by Collins (1966). Also Lorenz (1984) has obtained exact Kantowski-Sachs vacuum models in Brans-Dicke (1961) theory, while Singh and Agrawal (1991) discussed Kantowski-Sachs type models in Saez and Ballester (1985) scalar tensor theory. Adhav *et al*.(2008) have studied Kantowski-Scach cosmological model in general theory of Relativity.

Topological defects arise whenever symmetry is spontaneously broken. When the topology of the vacuum manifold exhibits disconnected regions we are facing domain walls this comes out which are defects arising from a breaking of a discrete symmetric group by means of a Higgs

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field Kibble (1976), Shellard and Vilenkin (1994), Cvetiv and Soleny (1997). From the gravitational point of view, an interesting feature of the walls gravitational field is that its weak field approximation does not corresponds to any exact static solution of the Einstein equations, hence implying that they are gravitationally unstable vilenkin (1981). Peter (1996), the internal structure of a surface current-carying wall was studied and the internal quantities such as the energy per unit surface and the surface current were calculated numerically. Vilenkin (1983), Ipser and Sikivie (1984), a time-dependent metric was obtained and it was shown that observers experience a repulsion from the walls. Current-carrying walls and their cosmological consequences were also objects of investigations.

The domain walls have become important in recent years from cosmological stand point when a new scenario of galaxy formation has been proposed by Hillet *et al* (1989). According to them the formation of galaxies are due to domain walls produced during phase transitions after the time of recombination of matter and radiation. So far a considerable amount of work has been done on domain walls. Widrow (1989), Goets (1990), Mukherjee (1993), Wang (1994), Rahaman et al (2001), Rahaman (2002), Reddy & Subbarao (2006) are some of the authors who have investigated several aspects of domain walls. Adhav *et al*.(2007) have investigated several aspects of strings and domain walls for Bianchi type space times.

In this paper, we have obtained Kantowski-Scach cosmological model in presence of domain walls include normal matter by using equation of state. Some physical and kinematical properties of the cosmological models are also discussed.

2. Field equations and their solutions

The study of domain-walls in general relativity has led to some rather intriguing results. Following Zel'dovich, Kobzarev and Okun (1974), if we consider a Lagrangean of a scalar field ϕ in the form

$$£=\frac{1}{2}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}-\lambda^2\left(\phi^2-n^2\right)^2 \quad , \tag{1}$$

then the classical field equations are

$$g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}-\left(\frac{\partial}{\partial\phi}\right)\left[\lambda^2\left(\phi^2-n^2\right)^2\right]=0 \tag{2}$$

In this broken-symmetry state, one may have two regions where $\phi=\pm n$ separated by a layer whose thickness at rest is small but non-vanishing. Obviously the energy-stress tensor vanishes in the region,where ϕ has the constant value $\pm n$. However, in the intervening layer, called the domain walls, it is not so. To evaluate the energy-stress tensor in the region, one supposes that

$\phi_{,\alpha}$ in the transition layer is a space like vector but Okyama N. and Maeda K. (2004) given by the energy-momentum tensor of the domain wall as

$$T^{D}{}_{ab} = (\rho + p)u_a u_b - p g_{ab} \quad , \tag{3}$$

Where u_i is the four velocity.

The energy-momentum tensor of the domain walls include normal matter described by ρ_m and p_m as well as a domain walls tension σ

$$\rho = \rho_m + \sigma \text{ and } p = p_m - \sigma \tag{4}$$

Also, ρ_m and p_m are related by the equation of state

$$p_m = (\gamma - 1)\rho_m \quad , \quad \text{where, } 1 \le \gamma \le 2$$

We consider the Kantowski-Sachs space time in the form

$$ds^2 = dt^2 - R^2 dr^2 - S^2 (d\theta^2 + \sin^2\theta \, d\phi^2) \ , \tag{5}$$

where *R* and *S* are the functions of time *'t'* only.

Einstein field equation can be expressed as

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G T_{ij} \quad . \tag{6}$$

Here we use gravitational units $8\pi G = C = 1$.

Now, with the help of equations (3) and (4) the filed equation (6) for the metric (5) can be written as

$$2\frac{S_{44}}{S} + \left(\frac{S_4}{S}\right)^2 + \frac{1}{S^2} = -p_m + \sigma \tag{7}$$

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4 S_4}{RS} = -p_m + \sigma \tag{8}$$

$$2\frac{R_4 S_4}{RS} + \left(\frac{S_4}{S}\right)^2 + \frac{1}{S^2} = \rho_m + \sigma \tag{9}$$

Here suffix '4' after an unknown function denotes partial differentiation with respect to *t*.

There are three total field equations with five unknowns, which are $R, S, p_{m,} \rho_m$ and σ. To get solutions of the field equation, we use equation of state

$$p_m = (\gamma - 1)\rho_m , \qquad 1 \le \gamma \le 2$$

When $\gamma = 1$ then $\rho_m = 0$, $\rho_m = 0$

Therefore, the set of equations (7) to (9) becomes

$$2\frac{S_{44}}{S}+\left(\frac{S_4}{S}\right)^2+\frac{1}{S^2}=\sigma \tag{10}$$

$$\frac{R_{44}}{R}+\frac{S_{44}}{S}+\frac{R_4S_4}{RS}=\sigma \tag{11}$$

$$2\frac{R_4S_4}{RS}+\left(\frac{S_4}{S}\right)^2+\frac{1}{S^2}=\sigma \tag{12}$$

Solving the field equations (10) to (12), an exact solution is obtain

$$R=M\ (\alpha t+\beta)^{\frac{n}{1-n}},\quad S=N(\alpha t+\beta\ )^{\frac{1}{1-n}}\quad , \tag{13}$$

where $M\ =(1-n)^{\frac{n}{1-n}}$ and $N=(1-n)^{\frac{1}{1-n}}$.

Domain walls tension $\sigma=\frac{n\alpha^2(2n+1)}{(1-n)^2}\frac{1}{T^2}$

Using (13), equation (5) becomes

$$ds^2=dt^2-M^2(\alpha t+\beta)^{\frac{2n}{1-n}}dr^2-N^2(\alpha t+\beta)^{\frac{2}{1-n}}(d\theta^2+\sin^2\theta\,d\phi^2)$$

Through a proper choice of co-ordinates and constants, Kantowski-Scach domain walls model can be written as

$$ds^2=-\frac{dT^2}{\alpha^2}-M^2(T)^{\frac{2n}{1-n}}dr^2-N^2(T)^{\frac{2}{1-n}}(d\theta^2+\sin^2\theta d\phi^2), \tag{14}$$

3. Physical Properties of the model

The physical quantities that are important in cosmology are proper volume V^3, expansion scalar θ, shear scalar σ^2 as follows

$$V^3=\sqrt{-g}=\frac{MN^2}{2}(T)^{\frac{n+1}{1-n}}$$

$$\theta=\frac{1}{3}U^j_{;i}$$

$$=\frac{1}{3}\frac{(n+2)\alpha}{1-n}\frac{1}{T}$$

$$\sigma^2=\frac{1}{2}\sigma^{ij}\sigma_{ij}$$

$$=\frac{1}{18}\frac{(n+2)^2}{(1-n)^2}\frac{\alpha^2}{}\frac{1}{T^2}$$

It may be observed that at initial moment. When T=0, the proper volume will be zero while domain wall tension diverge. When $T\to 0$, then expansion scalar θ, shear scalar σ^2 tends to ∞.

For large values of $T(T \to \infty)$, we observe that expansion scalar θ, shear scalar σ^2 becomes zero.

4. Conclusion

In this paper, we have obtained Kantowski-Scach cosmological model in presence of domain walls include normal matter in Einstein general theory of gravitation by using equation of state. In equation of state , if $\gamma = 1$, we get only pressureless and charged matter solutions, that is, domain walls and pressure of the matter disappear.

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