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Abstract We consider anisotropic, homogeneous two-fluid plane symmetric cosmological models in higher dimensions. Here one fluid represents the matter content of the universe and another fluid is chosen to model the CMB (cosmic microwave background) radiation. The radiation and matter content of the universe are in interactive phase. Also we have discussed the behaviour of fluid parameters and kinematical parameters.

Keywords A plane symmetric metric · Two fluid · Higher dimensions

1 Introduction

Friedman-Robertson-Walker (FRW) spatially homogeneous and isotropic models are widely considered as good approximation of the present and early stage of the universe. In the two fluid model, one fluid represents the matter content of the universe and another fluid is chosen to model the CMB radiation. The discovery of 2–73 K isotropic cosmic microwave background radiation (CMBR) motivated many authors to investigate FRW model with a two fluid source [1–3]. According to the observation of high red shift type-IA super nova and supernova cosmology project [4–9], the universe is expanding with positive acceleration, implying the existence of the total negative pressure for the universe. As compared to the homogeneous and isotropic FRW models, Bianchi space-times provide spatially homogeneous and isotropic models of the universe.

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The Bianchi type cosmological models for perfect fluid are studied by Beesham [10], Chakraborty and Roy [11], Kalligas et al. [12], Arbab [13], Beesham et al. [14] and Kilinc [15], obtained solutions of Einstein's field equations with varying G and Λ. Bianchi type-I model with varying G and Λ was investigated by Vishwakarma [16].

Bianchi type VI_0 model with a two fluid source has been investigated by Coley and Dunn [17]. Pant and Oli [18] have presented two-fluid cosmological models in Bianchi II space-times with co-moving two fluid sources. In the presence and absence of G and Λ, two fluid Bianchi type-I models are studied by Oli [19].

Plane symmetric inflationary model has astrophysical interest since cosmological models play a vital role in the structure formation of the universe. At the present state of evolution, the universe, on the whole, is spherically symmetric and isotropic. But in its early stages of evolution, it could not have had such a smoothed out picture. So here we consider plane symmetry which is less restrictive than spherical symmetry and isotropy.

Prominent results obtained in the development of the super-string theory, therefore the study of higher dimensional physics is important. In the latest study of super-strings and super-gravity theories, Weinberg [20] studied the unification of the fundamental forces with gravity, which reveals that the space-time should be different from four. The string theories are discussed in 10-dimensions or 26-dimensions of space-time, since the concept of higher dimensions is not unphysical. Because of this, studies in higher dimensions inspired many researchers to enter into such a field of study to explore the hidden knowledge of the universe [21–27]. Adhav et al. [28] have studied two fluid Bianchi type-V Cosmological models in general relativity. Recently Katore et al. [29] have presented plane symmetric cosmological models with perfect fluid and dark energy, motivating with this work, we have studied two-fluid cosmological models in higher dimensions.

In the present paper, we have investigated plane symmetric space-time with two-fluid source in higher dimensions and also discussed the physical behavior of the corresponding two fluid models in detail. Our investigation is an extension of the work carried out by Mete et al. [30].

2 Field Equations

A plane-symmetric metric is given by

$$ds^2 = dt^2 - A^2\left(dx^2 + dy^2\right) - B^2 dz^2 - C^2 dw^2, \tag{1}$$

where A, B and C are functions of t only.

The Einstein's field equations for a two fluid source in natural unit (gravitational units) are written as

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi T_{ij}. \tag{2}$$

The energy momentum tensor for a two fluid source is given by

$$T_{ij} = T_{ij}^{(m)} + T_{ij}^{(r)}, \tag{3}$$

where $T_{ij}^{(m)}$ is the energy momentum tensor for matter field and $T_{ij}^{(r)}$ is the energy momentum tensor for radiation field [17] which are given by

$$T_{ij}^{(m)} = (p_m + \rho_m) u_i^m u_j^m - p_m g_{ij}, \tag{4}$$

$$T_{ij}^{(r)} = \frac{4}{3}\rho_r u_i^r u_j^r - \frac{1}{3}\rho_r g_{ij} \tag{5}$$

with

$$g^{ij} u_i^m u_j^m = 1, \qquad g^{ij} u_i^r u_j^r = 1. \tag{6}$$

The off diagonal equations of (2) together with energy conditions imply that the matter and radiation are both co-moving, we get,

$$u_i^{(m)} = (0,0,0,0,1), \qquad u_i^{(r)} = (0,0,0,0,1). \tag{7}$$

Using (1), (3), (4), (5) and (6), the field equations (2) reduces to

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} = 8\pi\left(-p_m - \frac{\rho_r}{3}\right), \tag{8}$$

$$2\frac{\ddot{A}}{A} + 2\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}^2}{A^2} + \frac{\ddot{C}}{C} = 8\pi\left(-p_m - \frac{\rho_r}{3}\right), \tag{9}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = 8\pi\left(-p_m - \frac{\rho_r}{3}\right), \tag{10}$$

$$2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}^2}{A^2} = 8\pi(\rho_m + \rho_r). \tag{11}$$

By comparing (8), (9), (10) and (11), we get

$$\frac{\dot{A}}{A} = \frac{\dot{B}}{B} = \frac{\dot{C}}{C} \tag{12}$$

solving (12), yields

$$A = B = C = at + k, \tag{13}$$

where a is arbitrary constant and k is constant of integration.

Using above, a plane symmetric metric (1) can be written as

$$ds^2 = dt^2 - (at+k)^2\left[dx^2 + dy^2 + dz^2 + dw^2\right].$$

3 Some Physical and Kinematical Properties

We assume the relation between pressure and energy density of matter field through the "gamma-law" equation of state which is given by

$$p_m = (\gamma - 1)\rho_m, \quad 1 \le \gamma \le 2.$$

We get energy density of matter, energy density of radiation and total energy density as

$$\rho_m = \frac{15a^2}{(4-3\gamma)(at+k)^2}, \tag{14}$$

$$\rho_r = \frac{-9a^2(2\gamma - 1)}{(4-3\gamma)(at+k)^2}, \tag{15}$$

$$\rho = \rho_m + \rho_r,$$
$$\rho = \frac{6a^2}{(at+k)^2}. \tag{16}$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{4}(H_1 + H_2 + H_3 + H_4),$$

where $H_1 = H_2 = \frac{\dot{A}}{A}$, $H_3 = \frac{\dot{B}}{B}$, $H_4 = \frac{\dot{C}}{C}$ are the directional Hubble Parameter in the direction of x, y, z and w axes respectively.

Case I: Dust Model. In order to investigate the physical behaviour of the fluid parameters, we consider the particular case of dust i.e. when $\gamma = 1$.

The scalar of expansion, shear scalar and deceleration parameter are given by

$$\theta = 4H = \frac{4a}{at+k},$$
$$\sigma^2 = \frac{32a^2}{9(at+k)^2}, \tag{17}$$
$$q = 0.$$

The density parameters for matter and radiation are given by

$$\Omega_m = \frac{15}{4},$$
$$\Omega_r = -\frac{9}{4},$$
$$\Omega = \Omega_m + \Omega_r = \frac{3}{2},$$

where Ω is the total density parameter.

Case II: Radiation Universe (when $\gamma = \frac{4}{3}$). On substituting $\gamma = \frac{4}{3}$, we get, the scalar of expansion, shear scalar and deceleration parameter as

$$\theta = 4H = \frac{4a}{at+k},$$
$$\sigma^2 = \frac{32a^2}{9(at+k)^2}, \tag{18}$$
$$q = 0$$

and the density parameters using $\gamma = \frac{4}{3}$ in (14) and (15) are given by

$$\Omega_m = \infty,$$
$$\Omega_r = \infty.$$

Case III: Hard Universe ($\gamma \in (\frac{4}{3}, 2)$ let $\gamma = \frac{5}{3}$). The scalar of expansion, shear scalar and deceleration parameter in hard universe are

$$\begin{aligned} \theta &= 4H = \frac{4a}{at+k}, \\ \sigma^2 &= \frac{32a^2}{9(at+k)^2}, \\ q &= 0. \end{aligned} \tag{19}$$

For $\gamma = \frac{5}{3}$, (14) and (15) imply density parameters as

$$\begin{aligned} \Omega_m &= -\frac{15}{4}, \\ \Omega_r &= \frac{21}{4}, \\ \Omega &= \Omega_m + \Omega_r = \frac{3}{2}. \end{aligned}$$

Here ρ_r and total density ρ are positive whereas ρ_m is negative.

Case IV: Zeldovich Universe ($\gamma = 2$). In this case, we get, the scalar of expansion, shear scalar and deceleration parameter as

$$\begin{aligned} \theta &= 4H = \frac{4a}{at+k}, \\ \sigma^2 &= \frac{32a^2}{9(at+k)^2}, \\ q &= 0. \end{aligned} \tag{20}$$

We get, the energy density of matter, energy density of radiation and total energy density as

$$\begin{aligned} \Omega_m &= -\frac{15}{8}, \\ \Omega_r &= \frac{27}{8}, \\ \Omega &= \frac{3}{2}. \end{aligned}$$

Here ρ_m is negative and total density ρ and ρ_r is positive.

4 Conclusion

In this paper we have presented anisotropic, homogeneous plane symmetric cosmological models in higher dimensions which have two fluids as the source of gravitational field. One fluid is a radiating field, modeling the cosmic microwave background while the other is a matter field, modeling the material content of the universe. Here we have discussed a law of variation for the mean Hubble parameter in five dimensional plane symmetric cosmological models that yields $q = 0$ which implies that these two fluid models are expanding with constant velocity.

The sign of deceleration, parameters q indicates whether the model accelerates or not. The positive sign of q (> 1) corresponds to decelerating model whereas the negative sign $(-1 < q < 0)$ indicates acceleration and $q = 0$ corresponds to expansion with constant velocity. We have studied four different cases viz.

(i) Dust model ($\gamma = 1$).
(ii) Hard universe $\gamma \in (\frac{4}{3}, 2)$.
(iii) Radiation universe ($\gamma = \frac{4}{3}$).
(iv) Zeldovich universe ($\gamma = 2$).

Here in all cases, we get, the ratio $(\frac{\sigma}{\theta})^2 = \frac{2}{9} \neq 0$.

Therefore, these models do not approach isotropy for large value of t. These models come out to be rotating as well as expanding ones, the rate of expansion decrease with time, which can be thought of as realistic models. It is interesting to note that in absence of fifth dimension our investigations resembles with the investigations of Mete et al. [30].

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References

1. Coley, A.A., Tupper, B.O.J.: J. Math. Phys. **27**, 406 (1986)
2. Davidson, W.: Mon. Not. R. Astron. Soc. **124**, 79 (1962)
3. McIntosh, C.B.G.: Mon. Not. R. Astron. Soc. **140**, 461 (1968)
4. Garnavich, P.M., et al.: Astrophys. J. **4493**, L53 (1998)
5. Perlumutter, S., et al.: Astrophys. J. **483**, 565 (1997)
6. Perlumutter, S., et al.: Astrophys. J. **391**, 51 (1998)
7. Perlumutter, S., et al.: Astrophys. J. **517**, 565 (1999)
8. Riess, A.G., et al.: Astron. J. **116**, 1009 (1998)
9. Schmidt, B.P., et al.: Astrophys. J. **507**, 46 (1998)
10. Beeshasm, A.: Gen. Relativ. Gravit. **26**, 159 (1994)
11. Chakraborty, S., Roy, A.: Astrophys. Space Sci. **253**, 205 (1997)
12. Kalligas, D., Wegson, P.S., Everitt, C.W.: Gen. Relativ. Gravit. **27**, 645 (1995)
13. Arbab, A.I.: Class. Quantum Gravity **20**, 93 (2003)
14. Beeshasm, A., Ghost, S.G., Lombart, R.G.: Gen. Relativ. Gravit. **32**, 471 (2000)
15. Kilinc, C.B.: Astrophys. Space Sci. **289**, 103 (2004)
16. Vishwakarma, R.G.: Class. Quantum Gravity **17**, 3833 (2000)
17. Coley, A.A., Dunn, K.: Astrophys. J. **348**, 26 (1990)
18. Pant, D.N., Oli, S.: Astrophys. Space Sci. **281**, 623 (2002)
19. Oli, S.: Astrophys. Space Sci. **314**, 89 (2008)
20. Weinberg, S.: Physics in Higher Dimensions. World Scientific, Singapore (1986)
21. Chodos, A., Detweller, S.: Phys. Rev. D **21**, 2167 (1980)
22. Lorentz-Petzold: Gen. Relativ. Gravit. **17**, 1189 (1985)
23. Ibanez, J., Verdaguer, E.: Phys. Rev. D **34**, 1202 (1986)
24. Gleiser, R.J., Diaz, M.C.: Phys. Rev. D **37**, 3761 (1988)
25. Banerjee, S., Bhui, B.: Mon. Not. R. Astron. Soc. **247**, 57 (1990)
26. Reddy, D.R.K., Venkateswara Rao, N.: Astrophys. Space Sci. **277**, 461 (2001)
27. Khandekar, K., Gaikwad, M.: Proc. Einstein Found. Int. **11**, 95 (2001)
28. Adhav, K.S., Borikar, S.M., Desale, M.S., Raut, V.B.: Int. J. Theor. Phys. **50**, 1846 (2011)
29. Katore, S.D., Adhav, K.S., Shaikh, A.Y., Sancheti, M.M.: Astrophys. Space Sci. **333**(1), 333–341 (2012)
30. Mete, V.G., Murade Ku, P.B., Umarkar Ku, V.M., Ugale Ku, M.R.: Sci. Rev. Chem. Commun. **2**(3), 497–502 (2012)