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ANISOTROPIC COSMOLOGICAL MODELS WITH PERFECT FLUID AND DARK ENERGY

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ABSTRACT

In this paper we present the Bianchi type-III cosmological models with binary mixture of perfect fluid and dark energy. The perfect fluid is obeying the equation of state $p = \gamma\rho$ with $\gamma \in [0,1]$ whereas, the dark energy density is considered to be either the quintessence or the Chaplygin gas. The exact solutions of the Einstein's field equations are obtained in quadrature form.

***Keywords**: Bianchi type-III space-time · Perfect Fluid · Dark energy.*

1. INTRODUCTION

The nature of the dark energy component of the universe [1-3] remain one of the deepest mysteries of cosmology. There is certainly no lack of candidates: cosmological constant, quintessence [4 - 6], Phantom energy [7]. In Einstein's general relativity, in order to have such acceleration, one needs to introduce a component to the matter distribution of the universe with a large negative pressure. This component is usually referred as dark energy [DE]. Astronomical observations indicate that our universe is flat and currently consists of approximately 2/3 dark energy and 1/3 dark matter. The nature of dark energy as well as dark matter is unknown and many radically different models have been proposed, such as, a tiny positive cosmological constant, quintessence [8 -11], the non-linear $F(R)$ models[12], and dark energy in brane worlds, among many others [13 - 27]. As mentioned before, the existence of the dark energy fluids comes from the observations of the accelerated expansion of the universe and the isotropic pressure cosmological models give the best fitting of the observations. Although [28] have suggested cosmological model with anisotropic and viscous dark energy in order to explain on anomalous cosmological observation in the cosmic microwave background (CMB) at the largest angles. An alternative model for the dark energy density was used by [29], where the authors suggested the use of some perfect fluid but obeying "exotic" equation of state. This type of matter is known as Chaplygin gas.

Bianchi type models are among the simplest models with anisotropic background which have been studied by several authors in an attempt to achieve better understanding of the observed small amount of anisotropy in the universe. The same models have also been used to examine the role of certain anisotropic sources during the formation of the large-scale structure, we see in the universe today. Some Bianchi cosmologies, for example, are natural hosts of large-scale magnetic fields and therefore their study can shed light on the implications of cosmic magnetism for galaxy formation. The simplest Bianchi family that contains the flat FRW universe as a special case are the type-III space-times. The Bianchi-type-III universe is generalization of the open universe in FRW cosmology. Hence, its study is important as dark energy models with non-zero curvature [30].

Recently, Bijan Saha [31], Singh and Chaubey [32] and Katore *et al* [33] have studied the binary mixture of perfect fluid and dark energy for Bianchi type-I, V, III space-time respectively.

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In this paper, we study the evolution of an initially anisotropic universe given by a Bianchi type-III space time and a binary mixture of a perfect fluid obeying the equation of state $p = \gamma\rho$ and a dark energy given by either a quintessence or a Chaplygin gas.

2. METRIC AND FIELD EQUATIONS

We consider the Bianchi type – III metric in form

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)e^{-2ax}dy^2 - C^2(t)dz^2, \tag{2.1}$$

where $A(t)$, $B(t)$, $C(t)$ are the functions of t only and a is a constant.

The energy momentum tensor of the source is given by

$$T_i^j = (\rho + p)u_i u^j - p\delta_i^j, \tag{2.2}$$

where u^i is the flow vector satisfying

$$g_{ij}u^i u^j = 1 \ . \tag{2.3}$$

Here ρ is the total energy density of a perfect fluid and /or dark energy, while p is the corresponding pressure. p and ρ are related by an equation of state $p = \gamma\rho$.

By using co-moving co-ordinate system, from (2.2), we obtain

$$T_0^0 = \rho, \quad T_1^1 = T_2^2 = T_3^3 = -p \tag{2.4}$$

The Einstein field equations for metric (2.1) equations (2.2), (2.3) and (2.4) reduces to

$$\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} - \frac{a^2}{A^2} = k\rho \tag{2.5a}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -kp \tag{2.5b}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -kp \tag{2.5c}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{a^2}{A^2} = -kp \tag{2.5d}$$

$$a\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \tag{2.5e}$$

Here k is the Einstein gravitational constant and over dot (**.**) denotes the differentiation with respect to t. From (2.5e), we have,

$$A = mB \tag{2.6}$$

Let V be the function of time t defined by

$$V = ABC \tag{2.7}$$

Subtracting (2.5c) from (2.5b), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{2.8}$$

Now from (2.7) and (2.8), we get,

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\frac{\dot{V}}{V}=0$$

which on integration give ,

$$\frac{A}{B}=d_1\exp\left(x_1\int\frac{dt}{V}\right), \tag{2.9}$$

Subtracting (2.5d) from (2.5c), we get,

$$\frac{d}{dt}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)+\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)\frac{\dot{V}}{V}=0$$

Integrating, we get,

$$\frac{B}{C}=d_2\exp\left(x_2\int\frac{dt}{V}\right), \tag{2.10}$$

Subtracting (2.5b) from (2.5d), we obtain,

$$\frac{C}{A}=d_3\exp\left(x_3\int\frac{dt}{V}\right), \tag{2.11}$$

where d_1,d_2,d_3,x_1,x_2 and x_3 are constants of integration.

In view of $V=ABC$, we find the following relation between the constants d_1,d_2,d_3,x_1,x_2,x_3 as

$$d_3=d_1d_2,\quad x_3=x_1+x_2$$

Using above results we write the scale factors $A(t)$, $B(t)$ and $C(t)$ in explicit form as,

$$A(t)=D_1V^{\frac{1}{3}}\exp\left(X_1\int\frac{dt}{V}\right), \tag{2.12}$$

$$B(t)=D_2V^{\frac{1}{3}}\exp\left(X_2\int\frac{dt}{V}\right), \tag{2.13}$$

$$C(t)=D_3V^{\frac{1}{3}}\exp\left(X_3\int\frac{dt}{V}\right), \tag{2.14}$$

where $D_i(i=1,2,3)$ and $X_i(i=1,2,3)$ satisfy the relations $D_1D_2D_3=1$ and

$X_1+X_2+X_3=0$.

Using (2.6) and (2.7), equations (2.12), (2.13) and (2.14) can be written as

$$A(t)=V^{\frac{1}{3}}, \tag{2.15a}$$

$$B(t)=DV^{\frac{1}{3}}\exp\left(X\int\frac{dt}{V}\right), \tag{2.15b}$$

$$C(t)=D^{-1}V^{\frac{1}{3}}\exp\left(-X\int\frac{dt}{V}\right), \tag{2.15c}$$

where $X_2=0,\quad X_1=X_3=X$ and $mD_2=1$, $\frac{D_1}{m}=\frac{1}{D_3}=D$, X and D are constants.

Now, adding (2.5b), (2.5c), (2.5d) and 3 times equation (2.5a), we get

$$\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\right)+2\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}\right)-\frac{2a^2}{A^2}=\frac{3k}{2}(\rho-p) \tag{2.16}$$

From (2.8) and (2.7), we obtain

$$\frac{\ddot{V}}{V}-\frac{2a^2}{V^{\frac{2}{3}}}=\frac{3k}{2}(\rho-p)\ . \tag{2.17}$$

The conservational law for energy-momentum tensor gives

$$\dot{\rho}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)(\rho+p)=0\ . \tag{2.18}$$

From (2.18) and (2.7), we get

$$\dot{\rho}=-\frac{\dot{V}}{V}(\rho+p)\ . \tag{2.19}$$

From (2.17) and (2.19), we have

$$\dot{V}=\pm\sqrt{2\left(\frac{3k}{2}\rho V^2+\frac{3}{2}a^2V^{\frac{4}{3}}+c_1\right)}\ , \tag{2.20}$$

where c_1 is the integration constant.

Rewriting (2.19), in the form

$$\frac{\dot{\rho}}{(\rho+p)}=-\frac{\dot{V}}{V}\ . \tag{2.21}$$

We know that the pressure and the energy density obeys an equation of state of type $p=f(\rho)$.

We conclude that ρ and p are function of V. Hence, the right hand side of (2.17) is a function of V.

Now (2.17), can be rewritten as

$$\ddot{V}=\frac{3k}{2}(\rho-p)V+2a^2V^{\frac{1}{3}}\equiv F(V)\ . \tag{2.22}$$

From the mechanical point of view equation (2.22) can be interpreted as equation of motion of a single particle with unit mass under the force $F(V)$. Then

$$\dot{V}=\pm\sqrt{2(\in-U(V))}\,. \tag{2.23}$$

Here $\in$ can be viewed as energy and $U(V)$ is the potential energy of the force F.

Comparing (2.20) and (2.23), we find

$$\in=c_1$$

and

$$U(V)=-\left(\frac{3k}{2}\rho V^2+\frac{3}{2}a^2V^{\frac{4}{3}}\right). \tag{2.24}$$

Finally, we write the solution to (2.20) in quadrature form

$$\int \frac{dV}{\sqrt{2\left(\frac{3k}{2}\rho V^2+\frac{3}{2}a^2V^{\frac{4}{3}}+c_1\right)}}=t+t_0 \ , \tag{2.25}$$

where the integration constant t_0 can be taken to be zero, since it only gives a shift in time.

3. UNIVERSE AS A BINARY MIXTURE OF PERFECT FLUID AND DARK ENERGY

In this section we thoroughly study the evolution of the Binachi type-III universe filled with perfect fluid and dark energy in details

$$\rho=\rho_{pF}+\rho_{DE}, \quad p=P_{pF}+p_{DE} \ . \tag{3.1}$$

The energy momentum tensor can be decomposed as

$$T_i^{\,j}=\left(\rho_{DE}+\rho_{pF}+p_{DE}+p_{pF}\right)u_iu^j-\left(p_{DE}+p_{pF}\right)\delta_i^{\,j} \ . \tag{3.2}$$

In the above equation ρ_{DE} is the dark energy density, p_{DE} is the dark energy pressure. ρ_{pF} and p_{pF} denotes the energy density and pressure of the perfect fluid, respectively.

The perfect fluid obeys the equation of state

$$P_{pF}=\gamma\rho_{pF} \ , \tag{3.3}$$

where γ is the constant and lies in the interval $\gamma\in[0,1]$.

Depending on numerical value of γ, it describes the following types of universes

$$\gamma=0 \quad \text{(dust universe)}, \tag{3.4a}$$

$$\gamma=\frac{1}{3} \quad \text{(radiation universe)}, \tag{3.4b}$$

$$\gamma\in\left(\frac{1}{3},1\right) \quad \text{(hard universe)}, \tag{3.4c}$$

$$\gamma=1 \quad \text{(Zeldovich universe or stiff matter)} \ . \tag{3.4d}$$

In a co-moving frame the conservation law of energy momentum tensor leads to the balance equation for the energy density,

$$\dot\rho_{DE}+\dot\rho_{pF}=-\frac{\dot V}{V}\left(\rho_{DE}+\rho_{pF}+p_{DE}+p_{pF}\right). \tag{3.5}$$

The dark energy supposed to interact with itself only and it is minimally coupled to the gravitational filed. As a result the evolution equation for the energy density decouples from that of the perfect fluid and from (3.5) we obtain two balance equations,

$$\dot\rho_{DE}+\frac{\dot V}{V}\left(\rho_{DE}+p_{DE}\right)=0 \ , \tag{3.6a}$$

$$\dot\rho_{pF}+\frac{\dot V}{V}\left(\rho_{pF}+p_{pF}\right)=0 \ . \tag{3.6b}$$

From (3.3) and (3.6b), we get

$$\rho_{pF}=\frac{\rho_0}{V^{1+\gamma}}, \quad p_{pF}=\frac{\rho_0\gamma}{V^{1+\gamma}} \ , \tag{3.7}$$

where ρ_0 is constant of integration.

In the absence of dark energy,

$$\int \frac{dV}{\sqrt{2\left(\frac{3k}{2}\rho_0 V^{1-\gamma}+\frac{3}{2}a^2V^{\frac{4}{3}}+c_1\right)}}=t \tag{3.8}$$

In the limit of high matter densities (γ=1) the equation for $V(t)$ is given by

$$\int \frac{dV}{\sqrt{2\left(\frac{3k}{2}\rho_0+\frac{3}{2}a^2V^{\frac{4}{3}}+c_1\right)}}=t \tag{3.9}$$

3.1 Case with a Quintessence

Let us consider the case when the dark energy is given by quintessence which obeys the equation of state

$$p_q=\omega_q\rho_q, \tag{3.10}$$

where constant ω_q varies between -1 and zero, i.e. $\omega_q \in [-1,0]$

The case $\omega_q=-1$ is nothing but the case of cosmological constant (Λ).

From (3.10) and (3.6a), we get

$$\rho_q=\frac{\rho_{0q}}{V^{1+\omega_q}}, \quad p_q=\frac{\omega_q\rho_{0q}}{V^{1+\omega_q}}, \tag{3.11}$$

where ρ_{0q} is an integration constant.

Now, the evolution equation (2.17) for V can be written as

$$\ddot{V}=\frac{3k}{2}\left[\frac{(1-\gamma)\rho_0}{V^{\gamma}}+\frac{(1-\omega_q)\rho_{0q}}{V^{\omega_q}}\right]+2a^2V^{\frac{1}{3}} \tag{3.12}$$

Equation (3.12) can be written in quadrature form as

$$\int \frac{dV}{\sqrt{2\left[\left(\frac{3k}{2}\rho_0V^{1-\gamma}+\rho_{0q}V^{1-\omega_q}\right)+\frac{3}{2}a^2V^{\frac{4}{3}}+c_1\right]}}=t+t_0 , \tag{3.13}$$

where t_0 is a constant of integration that can be taken to be zero. In the limit of high matter densities (γ=1) the general solution of the gravitational equations for a Bianchi type-III geometry cannot be expressed in an exact analytic form.

3.2 Case with Chaplygin gas

Let us now consider the case when the dark energy is represented by Chaplygin gas governed by equation of state

$$p_c=-\frac{A}{\rho_c} , \tag{3.14}$$

with A being a positive constant

Now, from (3.6a) and (3.14), we obtain

$$\rho_c=\sqrt{\frac{\rho_{0c}}{V^2}+A}, \quad p_c=\frac{-A}{\sqrt{\frac{\rho_{0c}}{V^2}+A}} , \tag{3.15}$$

with ρ_{0c} being an integration constant.

Now, the evolution equation (2.17) for V can be written as

$$\ddot{V} = \frac{3k}{2}\left[\frac{(1-\gamma)\rho_0}{V^{\gamma}} + \sqrt{\rho_{0c} + AV^2} + \frac{AV^2}{\sqrt{\rho_{0c} + AV^2}}\right] + 2a^2V^{\frac{1}{3}} \ . \tag{3.16}$$

The corresponding solution in quadrature now has the form

$$\int \frac{dV}{\sqrt{2\left[c_1 + \frac{3}{2}a^2V^{\frac{4}{3}} + \frac{3k}{2}\left(\rho_0 V^{1-\gamma} + \sqrt{\rho_{0c}V^2 + AV^4}\right)\right]}} = t \ , \tag{3.17}$$

where the second integration constant has been taken to be zero.

4.1 A particular cases

Let us consider $\gamma = 1/3$ (Radiation)

For $c_1 = 0$, (3.8) reduces to

$$\int \frac{dv}{\sqrt{3kvV^{2/3} + 3a^2V^{4/3}}} = t, \tag{4.1}$$

which gives

$$V = \left[\frac{a^2t^2}{3} - \frac{k}{a^2}\right]^{3/2}, \tag{4.2}$$

From (2.15) and (4.2) we get

$$A(t) = \left[\frac{a^2t^2}{3} - \frac{k}{a^2}\right]^{1/2}, \tag{4.3a}$$

$$B(t) = D\left[\frac{a^2t^2}{3} - \frac{k}{a^2}\right]^{1/2} \times \ \exp\left(\frac{-Xa\sqrt{3}}{k}\sqrt{\frac{t^2a^4}{t^2a^4 - 3k}}\right), \tag{4.3b}$$

$$C(t) = D^{-1}\left[\frac{a^2t^2}{3} - \frac{k}{a^2}\right]^{1/2} \times \ \exp\left(\frac{Xa\sqrt{3}}{k}\sqrt{\frac{t^2a^4}{t^2a^4 - 3k}}\right), \tag{4.3c}$$

where X and D are constants

From (3.7) and (4.2) we get

$$\rho = \rho_0\left[\frac{a^2t^2}{3} - \frac{k}{a^2}\right]^{-2}, \tag{4.4a}$$

and

$$\rho = \frac{\rho_0}{3}\left[\frac{a^2t^2}{3} - \frac{k}{a^2}\right]^{-2}, \tag{4.4b}$$

The Physical quantities of observational interest in cosmology are the expansion Scalarθ, the mean anisotropy parameter A, the shear scalar $\sigma^2$and the parameter q. They are defined as

$$\theta = 3H, \tag{4.5}$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta Hi}{H}\right)^2 \tag{4.6}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3}Hi^2 - 3H^2\right) = \frac{3}{2}AH^2 \tag{4.7}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1\ . \tag{4.8}$$

With the use of (4.5) – (4.8) we can express the Physical quantities as

$$\theta = \frac{3a^4 t}{a^4 t^2 - 3k} \tag{4.9}$$

$$A = \frac{18X^2}{a^2 t^2 (a^4 t^2 - 3k)} \tag{4.10}$$

$$\sigma^2 = \frac{27X^2 a^6}{(a^4 t^2 - 3k)^3} \tag{4.11}$$

$$q = \left(\frac{3k}{a^4}\right)\frac{1}{t^2}, \tag{4.12}$$

where X is constant. For large t, the model tends to be isotropic.

5. MODELS WITH CONSTANT DECELERATION PARAMETER

Case 1: Power- Law

Here we take

$$V = at^b, \tag{5.1}$$

where a and b are constants

From (2.15) and (5.1) we get

$A(t) = a^{1/3} t^{b/3}$, (5.2a)

$$B(t) = Da^{1/3} t^{b/3} exp\left(\frac{X}{a(1-b)} t^{1-b}\right), \tag{5.2b}$$

$$C(t) = D^{-1} a^{1/3} t^{b/3} exp\left(-\frac{X}{a(1-b)} t^{1-b}\right), \tag{5.2c}$$

where X and D are constants.

From (3.7) and (5.1), we have

$$\rho = \rho o a^{-(1+\gamma)} t^{-(1+\gamma)b}, \tag{5.3a}$$

and

$$\rho = \gamma \rho o a^{-(1+\gamma)} t^{-(1+\gamma)b} \quad . \tag{5.3b}$$

With the use of (4.5) – (4.8) we can express the physical quantities as

$$\theta = \frac{b}{t}, \tag{5.4}$$

$$A = \frac{6X^2}{a^2 b^2} \frac{1}{t^{2(b-1)}}, \tag{5.5}$$

$$\sigma^2 = \frac{6X^2}{a^2} \frac{1}{t^{2b}}, \tag{5.6}$$

$$q = \frac{3}{b} - 1, \tag{5.7}$$

where X is constant. For large t, the model tends to be isotropic

Case II: Exponential – Type

Here we take

$$V = \alpha e^{\beta t}, \tag{5.8}$$

where α and β constants

From (2.15) and (5.8), we get

$$A(t) = \alpha^{1/3} exp\left(\frac{\beta t}{3}\right) \tag{5.9a}$$

$$B(t) = D\alpha^{1/3} exp\left(\frac{\beta t}{3} - \frac{X}{\alpha\beta} e^{-\beta t}\right) \tag{5.9b}$$

$$C(t) = D^{-1}\alpha^{1/3} exp\left(\frac{\beta t}{3} + \frac{X}{\alpha\beta} e^{-\beta t}\right) \tag{5.9c}$$

where *x* and D are constants

From (3.7) and (5.8), we get

$$\rho = \rho o \alpha^{-(1+\gamma)} e^{-(1+\gamma)\beta t}, \tag{5.10a}$$

and

$$P = \gamma \rho o \alpha^{-(1+\gamma)} e^{-(1+\gamma)\beta t} . \tag{5.10b}$$

With the use of (4.5) – (4.8), the physical quantities can be express as

$$\theta = \beta, \tag{5.11}$$

$$A = \frac{6X^2}{\alpha^2\beta^2} e^{-2\beta t}, \tag{5.12}$$

$$\sigma^2 = \frac{X^2}{\alpha^2} e^{-2\beta t} \tag{5.13}$$

$$q = -1, \tag{5.14}$$

where *X* is constant. For large *t*, the model tends to be isotropic

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6. CONCLUSION

In this paper we have studied the space-time geometry corresponding to Bianchi type-III filled with perfect fluid and dark energy in four dimensions. The exact solutions to the corresponding field equations are obtained in quadrature form. The inclusion of the dark energy into the system gives rise to an accelerated expansion of the model with initial singularities. Bianchi type-V cosmological model with perfect fluid and dark energy universe in has been investigated by Singh and Chaubey whose work has been extended and studied in Bianchi type-III in four dimensions. An attempt has been made to retain Singh and Chaubey forms of the various quantities. We have noted that all the results of Singh and Chaubey can be obtained from our results by assigning appropriate values to the function concerned.

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