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Bianchi Type-I Universe with Wet Dark Fluid in Bimetric Theory of Gravitation

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***ABSTRACT*:** The purpose of this paper is to investigate the role of wet dark fluid in Bianchi type-I cosmological model within the frame work of bimetric theory of gravitation proposed by Rosen (1973). We have used a new equation of state for the dark energy component of universe known as wet dark fluid given by $p_{WDF} = \gamma(\rho_{WDF} - \rho_*)$, which can describe a liquid for example water. In Bimetric theory of gravitation , it is observed that there is no contribution from wet dark fluid.

***KEYWORDS*:** Bianchi type I universe , Wet dark fluid , Rosen's bimetric theory.

I. INTRODUCTION

Einstein's theory of general relativity is one of the most beautiful structures of theoretical physics which describes the theory of gravitation in terms of geometry. In the last decades, several theories of gravitation have been proposed as alternatives to Einstein's theory of general relativity. The most popular amongst them is Rosen's (1973) bimetric theory of gravitation. In bimetric theory of gravitation the physical situation is determined by means of two metric tensors, i.e., the Riemannian metric tensor g_{ij} and the background metric tensor γ_{ij} corresponding to flat space-time. The Riemannian metric tensor g_{ij} plays the same role as in the Einstein's general relativity and it interacts with matter whereas the background metric tensor γ_{ij} is related to the geometry of the empty universe and describes the inertial forces. The interpretation of these two metric tensors in bimetric relativity theory is not unique. One can regard the γ_{ij} as a flat space time having no physical and geometrical significance but the physical metric tensor g_{ij} is considered as a gravitational potential tensor which is determined by field equations or by interactions between matter and gravitation. In the absence of matter, one should have $g_{ij} = \gamma_{ij}$. The bimetric theory of gravitation, like the Einstein general relativity theory, satisfies the covariance and equivalence principle.

Recently, Rosen (1978) proposed a new bimetric theory of gravitation on the cosmological basis in accordance with the perfect cosmological principle. In this new bimetric theory, the background metric tensor γ_{ij} is considered as describing a space time of constant curvature and not a flat space time of constant curvature but it should rather be chosen on the basis of cosmological consideration. Many authors Rosen (1975), Israelit (1976), Leibscher (1975), Karade (1980), Reddy and Venkateswarlu (1989), Reddy and Venkateshwara Rao (1998), Adhav et al (2002, 2005) and Katore

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et al (2004, 2006) have studied the bimetric theory of gravitation in different aspects with different space time.

Riess et al (1998), Perlmuttar et al (1998), Sahni (2004) studied the nature of the dark energy component of the universe as one of the deepest mysteries of cosmology. We are motivated to use the wet dark fluid (WDF) as a model for dark energy which stems from an empirical equation of state proposed by Tait (1988) and Hayward (1967) to treat water and aqueous solutions.

The equation of state for Wet Dark Fluid is

$$p_{WDF} = \gamma(\rho_{WDF} - \rho_*) \tag{1}$$

where the parameters γ and ρ_* are taken to be positive and we restrict ourselves to $0 \le \gamma \le 1$.

We have energy conservation equation as

$$\dot{\rho}_{WDF} + 3H(p_{WDF} + \rho_{WDF}) = 0. \tag{2}$$

Using equation of state and $3H = \dot{v}/v$ in the above equation , we get

$$\rho_{WDF} = \frac{\gamma}{1+\gamma}\rho_* + \frac{c}{v(1+\gamma)}, \tag{3}$$

where *c* is the constant of integration and *v* is the volume expansion.

WDF has two components : one behaves as cosmological constant and other as standard fluid with equation of state $p = \gamma\rho$.

If we take $c > 0$, this fluid will not violate the strong energy condition $p + \rho \ge 0$:

$$\begin{aligned} \mathrm{p}_{\mathrm{WDF}} + \rho_{WDF} &= (1+\gamma)\rho_{WDF} - \gamma\rho_* \\ &= (1+\gamma)\frac{c}{v(1+\gamma)} \ge 0. \end{aligned} \tag{4}$$

Holman and Naidu (2005) used the wet dark fluid as dark energy in the homogeneous, isotropic FRW case . Singh and Chaubey (2008) studied Bianchi type I universe with wet dark fluid.

In this paper, we study the Bianchi type-I cosmological model with wet dark fluid in bimetric theory of gravitation.

II. METRIC AND SOLUTIONS OF FIELD EQUATIONS

We consider Bianchi type I metric in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 , \tag{5}$$

where *A*, *B*, *C* are the functions of *t* only.

The background flat metric corresponding to equation (5) is

$$d\sigma^2 = dt^2 - dx^2 - dy^2 - dz^2 \tag{6}$$

The field equations in bimetric theory of gravitation proposed by Rosen (1973) are

$$N_j^i - \frac{1}{2} N\delta_j^i = -8\pi k T_j^i \tag{7}$$

Where $N_j^i = \frac{1}{2}\gamma^{ab}\left(g^{hi}g_{hj|a}\right)_{|b}$

and $k = \left(\frac{g}{r}\right)^{\frac{1}{2}}$

together with $g = \det(g_{ij})$ and $\gamma = \det(\gamma_{ij})$.

Here the vertical bar (|) denotes the covariant differentiation with respect to γ_{ij} and T_j^i is the energy momentum tensor of the matter field.

The Rosen's field equations in bimetric theory for the metric (5) can be written in the form

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 = -16\pi k T_1^1 \tag{8}$$

$$-\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 = -16\pi k T_2^2 \tag{9}$$

$$-\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = -16\pi k T_3^3 \tag{10}$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 16\pi k T_4^4 \tag{11}$$

Here the suffix 4 denotes differentiation with respect to *t.*

The energy momentum tensor (Singh and Chaubey, 2008) for the Wet Dark Fluid source is given by

$$T_j^i = \left(\rho_{WDF} + p_{WDF}\right)u_j u^i - p_{WDF}\delta_j^i \tag{12}$$

where u^i is the flow vector satisfying

$$g_{ij}u^i u^j = 1 \tag{13}$$

In commoving system of coordinates , From equations (12) & (13),we find

$$T_1^1 = T_2^{21} = T_3^3 = -p_{WDF} \quad ,T_4^4 = \rho_{WDF} \tag{14}$$

Now using equation (14) ,equations (8) to (11) becomes

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 = 16\pi k\, p_{WDF} \tag{15}$$

$$-\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 = 16\pi k p_{WDF} \tag{16}$$

$$-\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 16\pi k p_{WDF} \tag{17}$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 16\pi k \rho_{WDF} \tag{18}$$

From Equation (15) and (16), we obtain

$$\left(\frac{A_4}{A}\right)_4 = \left(\frac{B_4}{B}\right)_4 \tag{19}$$

From equation (15) and (17), we obtain

$$\left(\frac{A_4}{A}\right)_4 = \left(\frac{C_4}{C}\right)_4 \tag{20}$$

From equation (19) and (20), we obtain

$$\left(\frac{A_4}{A}\right)_4 = \left(\frac{B_4}{B}\right)_4 = \left(\frac{C_4}{C}\right)_4 \tag{21}$$

Adding equations (15) to (18), which admits the solution in the form

$$3p_{WDF} + \rho_{WDF} = 0 \tag{22}$$

For reality conditions the relations $p_{WDF} > 0$ and $\rho_{WDF} > 0$ must hold.

The above condition (22) is satisfied only when

$$p_{WDF} = 0 \quad \text{and} \quad \rho_{WDF} = 0$$

This means that the physical parameters, viz. wet dark fluid pressure (p_{WDF}) and wet dark fluid energy density (ρ_{WDF}), both, are identically zero. Thus, Bianchi type I universe with wet dark fluid in bimetric theory of gravitation does not survive and hence only vacuum model is obtained.

For vacuum case ($p_{WDF} = \rho_{WDF} = 0$), the field equations (11) – (14) admit the solution of the form

$$A = \exp(k_1 t + k_2) \;, \qquad B = \exp(k_3 t + k_4), \qquad C = \exp(k_5 t + k_6) \tag{23}$$

where k_1, k_2, k_3, k_4, k_5 and k_6 are the constant of integration.

Thus in view of equation (23), the metric (1) takes the form

$$ds^2 = dt^2 - \exp(k_1 t + k_2)^2 dx^2 - \exp(k_3 t + k_4)^2 dy^2 - \exp(k_5 t + k_6)^2 dz^2$$

Which [for $k_1 = k_3 = k_5 = \alpha$ and $k_2 = k_4 = k_6 = \beta$] reduces to

$$ds^2 = dt^2 - e^{(\alpha t + \beta)}\left(dx^2 + dy^2 + dz^2\right) \tag{24}$$

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III. CONCLUSION

Here, we have constructed Bianchi type-I cosmological model in Rosen's (1973) bimetric theory of gravitation with a new equation of state for the dark energy component of the universe(known as Wet Dark Fluid). It is observed that Bianchi type I cosmological model does not exist in bimetric theory of gravitation with wet dark fluid as source of gravitation and hence only vacuum model is obtained.

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