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# Einstein-Rosen Domain Walls in Saez-Ballester Theory of Gravitation

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***ABSTRACT*:** Field equations in the presence of domain walls for Einstein-Rosen cylindrically symmetric metric are obtained in a scalar-tensor theory of gravitation proposed by Saez-Ballester (*Phys.Lett.*A, **113,** 1985, 467). Static vacuum and non-static stiff fluid models are investigated. The physical and geometrical properties of the stiff fluid model are also studied.

***KEYWORDS*:** Domain walls, cylindrical symmetry, Scalar-tensor theory, Stiff fluid.

## I. INTRODUCTION

A scalar-tensor theory proposed by Saez-Ballester (1985) refers to a class of alternative theories of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies.

Cosmic strings and domain walls are the topologically stable objects which might have been formed during a phase transition in the early universe (Kibble, 1976). In particular, the appearance of domain wall is associated with spontaneously broken discrete symmetry i.e. the vacuum manifold consists of several disconnected components. For example, symmetry breaking is accomplished by a real scalar field. It is believed that the formation of galaxies is due to domain walls produced during the phase transition after the time of recombination of matter and radiation. Therefore, some recent works have focused on solutions to Einstein equations without cosmological constant describing the gravitational field of a planner wall i.e. a surface layer with plane symmetry. A vacuum domain wall in the thin-wall approximation is a planar wall in which the tension is equal to the surface energy density. After the time dependent solutions for domain walls considered by Vilenkin (1981,1983), Isper and Sikivie (1984) attempts were focused on trying to find the expansion in thick domain walls [Widrow(1989), Garfinkle *et*. *al*( 1990), Goetz (1990), and Mukherjee (1993)].

A considerable amount of work has been done on domain walls and cosmic strings in general relativity and in alternative theories of gravitation. In particular, Rahman , (2000), Reddy (2003a, b), Pradhan *et*. *al* (2003), Rahman and Mukherjee (2003), Reddy and Subba Rao (2006), Reddy *et*. *al*

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(2006, 2008) and Adhav *et. al* (2007) are some of the authors who have investigated various aspects of domain walls and cosmic strings in alternative theories of gravitation.

In this paper, we confine our work to the thick domain walls in a scalar-tensor theory (Saez-Ballester, 1985) for Einstein-Rosen cylindrically symmetric space-time. A static vacuum model and non-static stiff fluid models are presented. The physical and geometrical properties of the model are studied.

## II. THICK DOMAIN WALLS:

Thick domain walls can be viewed as a soliton-like solution of the scalar field equations coupled with gravity. In order to determine the gravitational field one has to solve Einstein field equations with an energy-momentum tensor $T_{ij}$ describing a scalar field $\phi$ with self-interactions contained in a potential $V(\phi)$ [Vilenkin (1983), Isper and Sikivie( 1984),Widrow (1989), and Goetz (1990)] is given by

$$T_{ij} = \phi_{,i}\,\phi_{,j} - g_{ij}\left(\frac{1}{2}\phi_{,k}\phi^{,k} - V(\phi)\right). \tag{1}$$

In second approach, to study the phenomenon, one has to assume the energy-momentum tensor in the form

$$T_{ij} = \rho(g_{ij} + W_i W_j) + pW_i W_j \tag{2}$$

with $W_i W^j = -1$,

where $\rho$ is the energy density of the wall, *p* is the pressure in the direction normal to the plane of the wall and $W_i$ is the unit space-like vector in the same direction (Rahman , Chakraborty and Kalam, 2001).

We consider the Einstein-Rosen cylindrically symmetric metric in the form

$$ds^2 = e^{2\alpha-2\beta}(dt^2 - dr^2) - r^2 e^{-2\beta} d\psi^2 - e^{2\beta} dz^2, \tag{3}$$

where $\alpha$ and $\beta$ are both the functions of *r* and *t* only. We denote the coordinates $r, \psi, z, t$ as $x^1, x^2, x^3, x^4$ respectively.

The field equations given by Saez-Ballester (1985) for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2} g_{ij} R - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\phi^{,k}\right) = T_{ij} \tag{4}$$

and the scalar field $\phi$ satisfies the equation

$$2\phi^n \phi^{i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0, \quad (5)$$

where $R_{ij}$ is the Ricci tensor, *R* is the scalar curvature, *n* is an arbitrary constant, $\omega$ is a dimensionless coupling constant and $T_{ij}$ is the matter energy-momentum tensor. Here comma and semicolon denote partial and covariant differentiation respectively (choose units so that $G = c = 1$).

The equations of motion

$$T^{ij}_{,j} = o \quad (6)$$

is a consequence of the field equations (4) and (5).

The energy-momentum tensor $T_{ij}$ ( in co-moving co-ordinates for thick domain walls) take the form

$$T_1^1 = -p, \quad T_2^2 = T_3^3 = T_4^4 = \rho, \quad T_4^1 = 0 \ \text{ and } \ T_j^i = 0 \ \text{ for } i \neq j. \quad (7)$$

The quantities $\rho$ and $p$ depend on $r$ and $t$ only.

Axial symmetry (assumed) implies that the scalar field $\phi$ shares the same symmetry as $\alpha$ and $\beta$.

As a consequence of which we note that

$$\phi_2 = \phi_3 = 0.$$

The field equations (4), (5) and (6) for the Einstein-Rosen metric with the help of (7) reduce to

$$e^{2\beta-2\alpha}\left[\beta_1^2 + \beta_4^2 - \frac{\alpha_1}{r} - \frac{\omega}{2}\phi^n(\phi_1^2 + \phi_4^2)\right] = p \ , \quad (8)$$

$$e^{2\beta-2\alpha}\left[\alpha_{11} - \alpha_{44} + \beta_1^2 - \beta_4^2 - \frac{\omega}{2}\phi^n(\phi_1^2 - \phi_4^2)\right] = \rho \quad (9)$$

$$e^{2\beta-2\alpha}\left[2\beta_{44} - 2\beta_{11} - \frac{2\beta_1}{r} + \alpha_{11} - \alpha_{44} - \beta_4^2 + \beta_1^2 - \frac{\omega}{2}\phi^n(\phi_1^2 - \phi_4^2)\right] = \rho \quad (10)$$

$$e^{2\beta-2\alpha}\left[\beta_1^2 + \beta_4^2 - \frac{\alpha_1}{r} - \frac{\omega}{2}\phi^n(\phi_1^2 + \phi_4^2)\right] = \rho \quad (11)$$

$$2\beta_1\beta_4 - \frac{\alpha_4}{r} - \omega\phi^n\phi_1\phi_4 = 0, \quad (12)$$

$$\phi_{11} + \frac{\phi_1}{r} + \frac{n}{2}\frac{\phi_1^2}{\phi} - \left[\phi_{44} + \frac{n}{2}\frac{\phi_4^2}{\phi}\right] = 0 \,, \tag{13}$$

$$p_1 + p(\alpha_1 - \beta_1) + \frac{p}{r} = 0 \tag{14}$$

and

$$\rho_4 + \rho(\alpha_4 - \beta_4) = 0 \,, \tag{15}$$

where and after words the subscript 1 and 4 represent partial differentiation with respect to *r* and *t* respectively.

Equations (8) and (11) immediately imply that

$$p = \rho \tag{16}$$

which shows that the solutions of the above field equations are possible only for **Stiff fluid (Zel'dovich fluid)**.

Since the field equations (8) - (15) are highly non-linear in nature , it is difficult to obtain the explicit solution in this case .Therefore, we consider only two physically interesting cases:

(i) Static vacuum $(p = \rho = 0)$ case (ii) Non-static stiff fluid ( $p = \rho$ ) case.

**II.1 Static vacuum model:**

Here we have $p = 0$ , $\rho = 0$ and $\alpha$ , $\beta$ , $\phi$ are the functions of *r* only.

In this case, the field equations (8)-(15) reduce to

$$\beta_1^2 - \frac{\alpha_1}{r} - \frac{\omega}{2}\phi^n\phi_1^2 = 0 \,,$$

$$\alpha_{11} + \beta_1^2 - \frac{\omega}{2}\phi^n\phi_1^2 = 0 \,,$$

$$\alpha_{11} - 2\beta_{11} + \beta_1^2 - \frac{2\beta_1}{r} - \frac{\omega}{2}\phi^n\phi_1^2 = 0$$

and

$$\phi_{11} + \frac{\phi_1}{r} + \frac{n}{2}\frac{\phi_1^2}{\phi} = 0 \quad .$$

Which admit the exact solutions given by

$$\alpha = c_1 \log r + c_2 \ , \qquad \beta = c_3 \log r + c_4$$

and

$$\phi^{\frac{n+2}{2}} = \left(\frac{n+2}{2}\right) c_5 \log r + \phi_0 \quad .$$

Where the constants $c_1, c_2, c_3, c_4, c_5$ and $\omega$ are related by equation

$$c_1 - c_3^2 + \frac{\omega}{2} c_5^2 = 0 .$$

After a suitable choice of co-ordinates and constant, Einstein-Rosen cylindrically symmetric metric (3) can be written as

$$ds^2 = r^{2A-2B}(dt^2 - dr^2) - r^{2(1-B)} d\psi^2 - r^{2B} dz^2 . \tag{17}$$

The scalar field $\phi$ in the universe is given by the relation

$$\exp\left\{ (\phi)^{\frac{n+2}{2}} \right\} = \phi_0 r^{c_5\left(\frac{n+2}{2}\right)} . \tag{18}$$

**II.2 Non-static stiff fluid model:**

Zel'dovich fluid(Stiff fluid) can be regarded as a perfect fluid having energy momentum tensor given by (2) and characterized by the equation of state (16).

According to Zel'dovich and Novikov(1971) ,it describes important cases e.g. radiation, relativistic degenerate Fermi gas and possible very dense matter.

Here we have $p = \rho$ and $\alpha$ , $\beta$ , $\phi$ are the functions of $t$ only.

In this case , the field equations (8)-(15) reduce to

$$e^{2\beta - 2\alpha}\left[ \beta_4^2 - \frac{\omega}{2}\phi^n \phi_4^2 \right] = p , \tag{19}$$

$$e^{2\beta - 2\alpha}\left[ \alpha_{44} + \beta_4^2 - \frac{\omega}{2}\phi^n \phi_4^2 \right] = -\rho , \tag{20}$$

$$e^{2\beta - 2\alpha}\left[ 2\beta_{44} - \alpha_{44} - \beta_4^2 + \frac{\omega}{2}\phi^n \phi_4^2 \right] = \rho \tag{21}$$

$$\frac{\alpha_4}{r} = 0 \tag{22}$$

and

$$\phi_{44} + \frac{n}{2}\frac{\phi_4^2}{\phi} = 0 \quad . \tag{23}$$

Which admit the exact solutions given by

$$\alpha = d_1 \;, \quad \beta = d_2 t + d_3 \;,$$

$$\phi = \left[\left(\frac{n+2}{2}\right)(d_4 t + d_5)\right]^{\frac{2}{n+2}}$$

And

$$p = \rho = d_6 e^{2\beta - 2\alpha} \quad ,$$

where the constants $d_i^{'s}$ and $\omega$ are related by

$$d_2^2 + d_6 - \frac{\omega}{2} d_4^2 = 0 \, . \tag{24}$$

After a suitable choice of co-ordinates and constant, cylindrically symmetric Einstein-Rosen metric (3) becomes

$$ds^2 = e^{-2T}(dT^2 - dr^2 - r^2 d\psi^2) - e^{2T} dz^2 \, . \tag{25}$$

This equation (25) represents a non-static Einstein-Rosen Zel'dovich universe in Saez-Ballester scalar-tensor theory of gravitation. It is free from initial singularity.

The scalar field in the universe is given by

$$\phi^{\frac{n+2}{2}} = \left(\frac{n+2}{2}\right) T \, . \tag{26}$$

The scalar field $\phi$, also, has no singularity.

According to Janis *et al* (1969) , Tabensky and Taub(1973), Rao and Tiwari(1979) another way to obtain these solutions is to know the solutions in the Einstein frame (or just use known scalar field solutions) and then perform a conformal transformation to the theory where the scalar field is non minimally coupled to gravity.

**III. Physical and geometrical properties:**

The pressure and energy density for the model (25) with the use of (24) is obtained as

$$p = \rho = \left(\frac{\omega - 2}{2}\right) e^{2T} \tag{27}$$

The physical and kinematical quantities for the model (25) have the following expressions:

Spatial volume : $V^3 = (-g)^{\frac{1}{2}} = re^{-2T}$ (28)

Expansion scalar : $\theta = u^i_{;i} = -e^T$ . (29)

Shear scalar: $\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{13}{6}e^{2T}$ . (30)

Deceleration parameter : $q = -3\theta^{-2}\left[\theta_{;\alpha}u^\alpha + \frac{1}{3}\theta^2\right] = 2$ . (31)

The model (25) has no finite singularity. As $T \to \infty$ the spatial volume and the energy density tends to zero and infinity respectively. The model (25) is a contracting model since the scalar expansion is always negative. Here $\left(\frac{\sigma^2}{\theta^2}\right)$ does not vanish for large value of $T$ which implies that the model (25) anisotropic and does not approach isotropy. The deceleration parameter $q$ acts as an indicator of the existence of inflation of the model. If $q > 0$, the model decelerates in the standard way while $q < 0$ indicates inflation. Here the positive value of $q$ implies that the model (25) decelerates in the standard way. The Seaz-Ballester scalar field $\phi$ increases indefinitely as time increases.

## *III. CONCLUSION:*

We have studied the behavior of cylindrically symmetric Einstein-Rosen thick domain walls in the Saez-Ballester scalar-tensor theory of gravitation. In this case, we have obtained Einstein-Rosen cylindrically symmetric static vacuum model in presence of thick domain walls in the Seaz-Ballester scalar-tensor theory. Also, non-static, non-singular contracting cosmological model filled with stiff matter is obtained which decelerates in the standard way.

It is important to note that the results obtained in this paper are identical with the results obtained earlier by Reddy *et. al* (2006). Therefore, thick domain walls play similar role as perfect fluid when cylindrically symmetric Einstein-Rosen metric is studied in scalar-tensor theory of gravitation proposed by Saez-Ballester(1985).

## *REFERENCES*

[1] Adhav, K.S., Nimkar, A.S., Naidu,R.L., Astrophys Space Sci. **312,** 165-169 (2007).
[2] Garfinkle, D. and Gregory, R.: Phys. Rev. D **41**, 1889 (1990).
[3] Goetz, G.: J. Math. Phys. **31**, 2683 (1990).
[4] Ipser, J. and Sikivie, P.: Phys. Rev. D **30**, 712 (1984).
[5] Janis, A.I., Robinson, D.C., Wiricour, J.:Phys.Rev. **186**, 1729 (1969).
[6] Kibble, T.W.B.: J. Phys. A **9**, 1387 (1976).
[7] Mukherjee, M.: Class. Quant. Grav. **10,** 131 (1993).
[8] Pradhan, A., Aotemshi, I. and Singh, G.P., Astrophys. Space Sci. **288**, 315 (2003).
[9] Rahaman, F., Mukherji,R.: Astrophys. Space Sci. **288**, 493 (2003).
[10] Rahman, F., Chakraborty ,S.and Kalam,M.: Int. J. Mod. Phys. D **10**, 735 (2001).
[11] Rahman, F.: Int. J. Mod. Phys. **D9**, 775 (2000).
[12] Rao, P.P., Tiwari, R.N.: Acta. Phys. Acad. Hung. **47**, 281 (1979).
[13] Reddy, D.R.K., Govinda Rao, P., Naidu, R.L., Int.J.Theor.Phys., **47** ,2966-2970 (2008).
[14] Reddy, D.R.K., Subba Rao, M.V.,Astrophys Space Sci. **302,**157-160 (2006).
[15] Reddy, D.R.K.,Adhav,K.S., Katore,S.D.: Astrophys Space Sci. **301**, 149 (2006).
[16] Reddy, D.R.K.: Astrophys Space Sci. **286**, 359 (2003a) .
[17] Reddy, D.R.K.: Astrophys Space Sci. **286**, 365 (2003b) .
[18] Saez , D., Ballester, V.J.: Phys. Lett. A**113**, 467 (1985).
[19] Tabensky,R.,Taub,A.H.:Commun.Math.Phys.**29**,61(1973).
[20] Vilenkin,A.: Phys. Lett. B **133** , 177 (1983).
[21] Vilenkin,A.: Phys. Lett. B **133** 177(1983).
[22] Vilenkin,A.: Phys. Rev D**24**, 2082(1981).
[23] Widrow,L.M.: Phys. Rev. D **39**, 3571 (1989).
[24] Zel'dovich,Ya.B.,Novikov,I.D.: Relativistic .Astrophysics,Vol.**6**,University of Chicago Press, Chicago (1971).