Article

Domain Wall Cosmological Model in Scalar Tensor Theory of Gravitation

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Abstract

An axially symmetric Bianchi Type-I domain wall cosmological model is obtained in scalar tensor theory of gravitation proposed by Saez and Ballester(1985). Some physical and kinematical properties of the model are discussed.

Keywords: Domain walls, axially symmetric Bianchi type-I, Saez-Ballester theory.

1. Introduction

In recent years there has been lot of interest in several alternative theories of gravitation. The most important among them are scalar-tensor theories of gravitation formulated by Brans and Dicke (1961), Nordvedt(1970) and Saez -Ballester(1985).All version of the scalar-tensor theories are based on the introduction of a scalar field ϕ into the formulation of general relativity, this scalar field together with metric tensor field then forms a scalar-tensor field representing the gravitational field.

The field equations given by Saez and Ballester(1985) for the combined
Scalar and tensor fields are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{1}$$

$$2\phi^n\phi^i_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor, R_{ij} is the Ricci tensor, R is the scalar curvature, n an arbitrary constant , ω is a dimensionless coupling constant and T_{ij} is the matter energy-momentum tensor. Here comma and semicolon denote partial and covariant differentiation respectively (we have chosen the units such that $8\pi G = 1 = C$).

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The equation of motion

$$T^{ij}_{;j} = 0 \quad , \tag{3}$$

is a consequence of field equation (1) and (2).

Domain walls are topological defects associated with spontaneous symmetry breaking whose plausible production site in cosmological phase transitions in the early universe, like others kinds of topological defects such as monopoles, cosmic string and textures. In general, domain walls are generated at rest in the very early universe at rest, with a curvature of the order of the characteristic scale at that moment. They are accelerated later if their interaction with other types of matter is small. When they are moving, their effective equation of state Fabris and Borba (2003) becomes $p = \left(\vec{v}^2 - \frac{2}{3}\right)\rho$.

Hence, the effective equation of state of a network of domain walls evolves from the rest case, $p = -\frac{2}{3}\rho$ to the relativistic case $p = \frac{\rho}{3}$ .When the domain walls reach the relativistic regime, their characteristic length becomes comparable to the Hubble radius. In many weak interacting domain walls model, the relativistic regime is reached when domain walls begin to dominate the matter content of the Universe Vilenkin and Shellard(1994).However, domain walls must feel a friction force, proportional to their velocity, due to their interaction with the other components of the matter content of the universe.

According to them the formation of galaxies are due to domain walls produced during phase transitions after the time of recombination of matter and radiation. A detailed discussion of Saez and Ballester cosmological model is contained in the work of Reddy and Venkateswara Rao(2001), Adhav *et al*. (2007), Ugale(2014) and Pund & Nimkar (2015) .Also, Reddy and Subbarao (2006) ,Adhav *et al*.(2007,10), katore *et al*(2011)and Pund & Avachar (2014) are some of the authors who have investigated several aspects of domain walls.

2. Metric and field equations

We consider axially symmetric Bianchi type-I metric

$$ds^2 = -dt^2 + A^2dx^2 + B^2\left(dy^2 + dz^2\right) \tag{4}$$

Where *A* and *B* are functions of time *t* alone.

A thick domain wall can be viewed as a solution like solution of the scalar field equations coupled with gravity. There are two ways of studying thick domain walls. One way is to solve gravitational field equations with an energy-momentum tensor describing a scalar field ψ with self interactions contained in a potential $v(\psi)$ given by

$$\psi_i \psi_j - g_{ij}\left[\frac{1}{2}\psi_k \psi^k - v(\psi)\right]. \tag{5}$$

Second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho(g_{ij} + \omega_i \omega_j) + p\ \omega_i \omega_j, \qquad \omega^i \omega_j = -1 \tag{6}$$

where ρ is the energy density of the walls, p is the pressure in the direction normal to the plane of the wall and ω_i is a unit space- like vector in the same direction. Here we use the second approach to study the thick domain walls in scalar tensor theory of gravitation.

The field equations (1-2) for the metric (4) with matter distribution (6) yield

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{\omega}{2}\phi^n \phi_4^2 = p \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{\omega}{2}\phi^n \phi_4^2 = -\rho \tag{8}$$

$$2\frac{A_4 B_4}{AB} + \left(\frac{B_4}{B}\right)^2 + \frac{\omega}{2}\phi^n \phi_4^2 = -\rho \tag{9}$$

$$\rho_4 + \rho\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right) = 0 \tag{10}$$

Here suffix ‘4’ after an unknown function denote partial differentiation with respect to *t*.

3. Solutions of the model

The set of field equation (7) to (10) are highly nonlinear in matter, in general , it is difficult to obtain the explicit solution of the field equation. The relation between metric coefficient *A* and B as well as *p* and ρ

Using these relation, an exact solution of the field equation are

$$A = k_5 (k_2 t + k_3)^{\frac{n}{k_1+1}}, \quad B = k_4 (k_2 t + k_3)^{\frac{1}{k_1+1}} \tag{11}$$

where, $k_4 = (k_1+1)^{\frac{1}{k_1+1}}$ and $k_5 = k_4^n$ and

$$\rho = k_{10}(k_2 t + k_3)^{\frac{-n-2}{k_1+1}} \qquad \text{where } k_{10} = \frac{k_9}{k_4^{n+2}} \tag{12}$$

$$p = k_{11}(k_2 t + k_3)^{\frac{-n-2}{k_1+1}} \qquad \text{where } k_{11} = 3k_{10} \tag{13}$$

The scalar field ϕ is given by

$$\phi=\left(\frac{n+2}{2}\right)^{\frac{2}{n+2}}\left[\left(\frac{k_1+1}{k_1-n-1}\right)k_7\left(k_2t+k_3\right)^{\frac{k_1-n-1}{k_1+1}}+k_8\right]^{\frac{2}{n+2}} \tag{14}$$

Through a proper choice of co-ordinates and constants the axially symmetric domain wall cosmological model using equation (13) can be written as

$$ds^2=-\frac{dT^2}{k_2{}^2}+k_5{}^2(T)^{\frac{2n}{k_1+1}}dx^2+k_4{}^2(T)^{\frac{2}{k_1+1}}\left(dy^2+dz^2\right) \tag{15}$$

where $T=\left(k_2t+k_3\right)$

4. Physical and Kinematical Properties

The physical and kinematical quantities for the model (15) have the following expression

$$\text{Proper volume}\,V^3=\sqrt{-g}=k_{12}(T)^{\frac{n+2}{k_1+1}} \tag{16}$$

$$\text{Expansion Scalar}(\theta)=\frac{k_{12}}{T},\quad k_{12}=\frac{(n+2)k_2}{3(k_1+1)} \tag{17}$$

$$\text{Shearscalar}\left(\sigma^2\right)=\frac{k_{13}}{T^2},k_{13}=\frac{7}{18}k_{12}^2 \tag{18}$$

The expression scalar field ϕ, pressure p and energy density ρ for the domain walls in Saez-Ballester theory are given by (12-14). It may be observed that at the initial moment, when $T=0$ the special volume will be zero. While energy density ρ and pressure p diverge. when $T\to 0$, then the expansion scalar θ and shear scalar σ^2 tends to ∞.
For large value that T we observed that the special volume, expansion scalar θ and shear scalar σ^2 becomes zero.

5. Conclusion

In this paper, we have consider Saez-Ballester field equation in the presence of domain walls . For solving field equation relation between metric potential are used. The cosmological model, thus obtained, represents a radiating universe in Saez-Ballester theory of gravitation.

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