Page No. : 43 - 53

SOME BIANCHI TYPE COSMOLOGICAL MODELS IN SCALE-COVARIANT THEORY OF GRAVITATION

K.S.Adhav V.G.Mete,A.S.Nimkar,A.M.Pund
Department of Mathematics,
S.G.B.Amravati University, Amravati – 444 602, India
E-mail : ati_ksadhav@yahoo.co.in

Abstract :

Non existance of Bianchi type-I and type-V cosmological models in scale-covariant theory of gravitation is shown when the source of gravitation is governed by either cosmic strings or domain walls.

Keywords :

Cosmic Strings, Domain walls, Scale-covariant theory, Bianchi Models.

Introduction :

Einstein's general theory of relativity has been successful in describing gravitational phenomena and served as a basis for models of the universe. However, since Einstein first published his theory of gravitation, there have been many criticism of general relativity because of the lack of certain 'desirable' features in the theory. For example, Einstein himself pointed out that general relativity does not account satisfactorily for inertial properties of matter. i.e. Mach's principle is not substantiated by general relativity. So in recent years there has been lot of interest in several alternative theories of gravitation. The most important among them are scalar-tensor theories of gravitation formulated by Brans-Dicke (1961), Nordt-Vedt (1970) and Saez – Ballester (1985). All versions of the scalar-tensor theories are based on the introduction of a scalar field ϕ into the formulation of general relativity. This scalar field together with the metric tensor field then forms a scalar-tensor field representing the gravitational field.

Canuto et al (1977) formulated a scale-covariant theory of gravitation which also admits a variable G and which is a viable alternative to general relativity. In the scale-covariant theory, Einstein's field equations are valid in gravitational units whereas physical quantities are measured in atomic units. The metric tensors in the two systems of units are related by a conformal transformation

$$\bar{g}_{ij} = \phi^2 (x^k) g_{ij} \tag{1}$$

where in Latin indices takes values 1,2,3,4, bars denote gravitational units and unbar denotes atomic quantities. The gauge function ϕ ($0<\phi<\infty$) in its most general formulation is a function of all space time coordinates. Thus, using the conformal transformation of the type given by (1) Canuto et al (1977) transformed the usual Einstein equation into

$$R_{ij} - \frac{1}{2} R g_{ij} + f_{ij}(\phi) = -8\pi G(\phi) T_{ij} + \Lambda(\phi) g_{ij} \tag{2}$$

where

$$\phi^2 f_{ij} = 2\phi\phi_{i;i} - 4\phi_i\phi_j - g_{ij}(\phi\phi^k_{;k} - \phi^k\phi_k) \tag{3}$$

where R_{ij} is the Ricci tensor, R the Ricci scalar, Λ the cosmological constant, G the gravitational 'constant' and T_{ij} is the energy momentum tensor. A semicolon denotes covariant derivative and ϕ_i denotes ordinary derivative w.r.to x^i. A particular feature of this theory is that no independent equation for ϕ exists. The possibilities that have been considered for gauge functions ϕ are

$$\phi(t) = \left(\frac{to}{t}\right)^{\epsilon}, \qquad \epsilon = \pm 1, \ \pm\frac{1}{2}. \tag{4}$$

where t_o is constant. The form $\phi \sim t^{\frac{1}{2}}$ is the one most favoured to fit observations [ ].

The energy conservation equation for perfect fluid

$$\rho_4 + (\rho + p)u^i_{;i} = -\rho\frac{(G\phi)_4}{G\phi} - 3p\frac{\phi_4}{\phi} \tag{5}$$

A detailed discussion of scale covariant theory is contained in the work of Canuto et al (1977 b), Beeshan (1986 a,b,c), Reddy & Venkateswarlu (1987), Reddy et al (2002) and Reddy and Venkateswarlu (2004) have investigated several aspects of this theory of gravitation with the perfect fluid matter distribution as source.

It is still challenging problem to know the exact physical situation at very early stages of the formation of our universe. At the very early stages of evolution of the universe, it is generally assumed that during the phase transition (as the universe passes through its critical temperatures) the symmetry of the universe is broken spontaneously. It can give rise to topological stable defects such as strings, domain walls and monopoles (Kibble, 1976) of all these cosmological structures, cosmic strings and domain walls have excited the most interest. The gravitational effects of cosmic strings, both in general relativity and in the alternative theories of gravitation, have been extensively discussed by Vilenkin (1981), Gott (1985), Letelier (1983), Stachel (1980), Krori et.al (1990), Banerjee et al (1990), Tikekar and Patel (1990), Tikekar et al (1994), Rahaman et al (2002), Reddy (2003 a,b), Reddy (2005 a,b) and Adhav et al (2007 a,b).

In particular, the domain walls have become important in recent years from cosmological stand point when a new scenerio of galaxy formation has been proposed by Hillet et al (1989). According to them the formation of galaxies are due to domain walls produced during phase transitions after the time of recombination of matter and radiation. So far a considerable amount of work has been done on domain walls. Vilenkin (1983), Ipser & Sikivie (1984), Widrow (1989), Goets (1990), Mukherjee (1993), Wang (1994), Rahaman et al (2001), Rahaman (2002), Reddy & Subbarao (2006) are some of the authors who have investigated several aspects of domain walls.

The purpose of the present work is to study Bianchi type-I and type-V cosmological models in a scale covariant theory of gravitation with cosmic strings & domain walls. Our paper is orgnized as follows. In section 2, we discuss Bianchi type – I and type-V strings cosmological models in the scale-covariant theory of gravitation., In section 3, we discuss the thick domain walls in Bianchi type-I and type-V space time. The last section contains conclusions.

2. Cosmic Strings

In this section, we discuss the non-existence of Bianchi type -I and type-V cosmic strings in the scale-covariant theory of gravitation. Here we consider the energy momentum tensor for cosmic string source as

$$T_j^i = \rho\, u^i u_j - \lambda x^i x_j \tag{6}$$

where ρ is the rest energy density of the cloud of strings with massive particles attached to them. $\rho=\rho_p+\lambda$, ρ_p being the rest energy of the particles attached to the strings & λ the tension density of the system of strings. As pointed out by Letelier (1983), λ may be positive or negative, u^i describes the cloud four-velocity and x^i represents the direction of strings.

2.1 Bianchi type – I space - time

We consider the Bianchi type-I metric given by (Reddy 2003)

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 \quad , \tag{7}$$

where A and B are functions of 't' only. Orthonormalism of u^i and x^i is given as

$$u^i u_i = 1, \quad x^i x_j = 0, \quad x^i x_i = -1 \tag{8}$$

In the comoving coordinate system, we have from (6)

$$T_1^1 = T_2^2 = 0, \quad T_3^3 = \lambda, \quad T_4^4 = \rho, \;\; T_j^i = 0 \qquad \text{for } i \neq j \tag{9}$$

The quantities ρ and λ depend on t only. Here the string source is along z-axis which is the axis of symmetry.

Now, with the help of (6), (8) & (9) the field equations (2), (3) & (5) for the metric (7), with zero cosmological 'constant' can be written as

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}-\frac{A_4\phi_4}{A\phi}+\frac{B_4\phi_4}{B\phi}+\frac{C_4\phi_4}{C\phi}+\frac{\phi_{44}}{\phi}-\left(\frac{\phi_4}{\phi}\right)^2=0 \tag{10}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}+\frac{A_4\phi_4}{A\phi}-\frac{B_4\phi_4}{B\phi}+\frac{C_4\phi_4}{C\phi}+\frac{\phi_{44}}{\phi}-\left(\frac{\phi_4}{\phi}\right)^2=0 \tag{11}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}+\frac{A_4\phi_4}{A\phi}+\frac{B_4\phi_4}{B\phi}-\frac{C_4\phi_4}{C\phi}+\frac{\phi_{44}}{\phi}-\left(\frac{\phi_4}{\phi}\right)^2=8\pi G\lambda \tag{12}$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}+\frac{A_4\phi_4}{A\phi}+\frac{B_4\phi_4}{B\phi}+\frac{C_4\phi_4}{C\phi}-\frac{\phi_{44}}{\phi}+3\left(\frac{\phi_4}{\phi}\right)^2=8\pi G\rho$$
(13)

$$\rho_4+(\rho+\lambda)\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=-\rho\frac{G_4}{G}-(\rho+3\lambda)\frac{\phi_4}{\phi} \tag{14}$$

where the suffix 4 after an unknown function denotes differentiation w.r. to '*t*'.

The field equations (10) – (13) are four equations in seven unknowns *A, B, C,* ϕ, ρ, λ and $G(\phi)$.

Hence to get a determinate solution we assume a relation between metric potential

$A = \alpha C,\qquad \alpha =$ constant (15)

Using equation (15), the sets of equations (10) – (14) reduces to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}+\frac{B_4\phi_4}{B\phi}+\frac{\phi_{44}}{\phi}-\left(\frac{\phi_4}{\phi}\right)^2=0 \tag{16}$$

$$2\frac{C_{44}}{C}+\left(\frac{C_4}{C}\right)^2-\frac{B_4\phi_4}{B\phi}+\frac{\phi_{44}}{\phi}-\left(\frac{\phi_4}{\phi}\right)^2=0 \tag{17}$$

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}+\frac{B_4\phi_4}{B\phi}+\frac{\phi_{44}}{\phi}-\left(\frac{\phi_4}{\phi}\right)^2=8\pi G\lambda \tag{18}$$

$$2\frac{B_4C_4}{BC}+\left(\frac{C_4}{C}\right)^2+2\frac{C_4\phi_4}{C\phi}+\frac{B_4\phi_4}{B\phi}-\frac{\phi_{44}}{\phi}+3\left(\frac{\phi_4}{\phi}\right)^2=8\pi G\rho \tag{19}$$

$$\rho_4+(\rho+\lambda)\left(\frac{B_4}{B}+2\frac{C_4}{C}\right)=-\rho\frac{G_4}{G}-(\rho+3\lambda)\frac{\phi_4}{\phi} \tag{20}$$

From equation (16) & (18),

we get $\lambda = 0$ (21)

In the literature (Letelier 1983), we have the equations of state for strings model as

$\rho = \lambda$ (geometric or Nambu String) (22)

$\rho = (1+\omega)\lambda$ (p - string or Takabayasi String) (23)

$\rho + \lambda = 0$ (Reddy String) (9,36,37) (24)

Using equations (21) in (22), (23) & (24), we get $\rho = 0$, which shows that in scale covariant theory neither geometric string nor p-string nor Reddy string survive. Hence we observe that the geometric strings, p-strings and Reddy strings do not exists in the scale covarient theory of gravitation.

2.2 Bianchi type – V space - time

We consider the Bianchi type-V metric given by

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x}(dy^2 + dz^2) \quad , \tag{25}$$

where A and B are functions of 't' only. Orthonormalism of u^i and x^i is given as

$$u^i u_i = -1, \quad x^i x_j = 0, \quad x^i x_i = 1 \tag{26}$$

In the comoving coordinate system, we have from (6)

$$T_1^1 = -\lambda, \quad T_2^2 = T_3^3 = 0, \quad T_4^4 = -\rho, \tag{27}$$

The quantities ρ and λ depend on t only. Here the string source is along x-axis which is the axis of symmetry.

Now with the help of (6), (26) & (27) the field equations (2), (3) & (5) for the metric (25), with zero cosmological 'constant' can be written as

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{1}{A^2} - \frac{A_4\phi_4}{A\phi} + 2\frac{B_4\phi_4}{B\phi} + \frac{\phi_{44}}{\phi} + \left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\lambda \tag{28}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} + \frac{A_4\phi_4}{A\phi} + \frac{\phi_{44}}{\phi} + \left(\frac{\phi_4}{\phi}\right)^2 = 0 \tag{29}$$

$$2\frac{A_4 B_4}{AB} + \left(\frac{B_4}{B}\right)^2 - \frac{3}{A^2} = 8\pi G\rho \tag{30}$$

$$\frac{A_4}{A} = \frac{B_4}{B} \tag{31}$$

where the suffix 4 after an unknown function denotes differentiation w.r. to 't'.

$$\rho_4 + (\rho + \lambda)\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right) = -\rho\frac{G_4}{G} - (\rho + 3\lambda)\frac{\phi_4}{\phi} \tag{32}$$

With the help of Equation (31), Equation (28) – (32) reduces to

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{1}{B^2} + \frac{B_4\phi_4}{B\phi} + \frac{\phi_{44}}{\phi} + \left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\lambda \tag{33}$$

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{1}{B^2} + \frac{B_4\phi_4}{B\phi} + \frac{\phi_{44}}{\phi} + \left(\frac{\phi_4}{\phi}\right)^2 = 0 \tag{34}$$

$$3\left(\frac{B_4}{B}\right)^2 - \frac{3}{B^2} = 8\pi G\rho \tag{35}$$

$$\rho_4 + 3(\rho + \lambda)\frac{B_4}{B} = -\rho\frac{G_4}{G} - (\rho + 3\lambda)\frac{\phi_4}{\phi} \tag{36}$$

Solving above equation of From equation (33) & (34),

we get , $\lambda = 0$ (37)

Using equation (37) in (22), (23) & (24),

we get $\rho = 0$

Here we again observe that the geometric strings, p-strings and Reddy strings do not exist in the scale covarient theory of gravitation.

3. Thick Domain Walls :

In this section we discuss the thick domain walls in the Bianchi type-I and type-V space-time given by (7) & (25). A thick domain wall can be viewed as a solution like solution of the scalar field equations coupled with gravity. There are two ways of studying thick domain walls. One way is to solve gravitational field equations with an energy-momentum tensor describing a scalar field ψ with self interactions contained in a potential $v(\psi)$ given by

$$\psi_i\psi_j - g_{ij}\left[\frac{1}{2}\psi_k\psi^k - v(\psi)\right]. \tag{38}$$

Second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho(g_{ij} + \omega_i\omega_j) + p\ \omega_i\omega_j, \qquad \omega^i\omega_j = -1 \tag{39}$$

where ρ is the energy density of the walls, p is the pressure in the direction normal to the plane of the wall and ω_i is a unit space- like vector in the same direction (32).

Here we use the second approach to study the thick domain walls in scale-covariant theory of gravitation.

3.1 Bianchi type-I space time

Consider the axially symmetric metric given by (7). In comoving coordinate system we have from (18)

$$T_1^1 = T_2^2 = T_4^4 = \rho, \quad T_3^3 = -p, \quad T_j^i = 0, \text{ for } i \neq j \tag{40}$$

Here pressure is taken in the direction of Z-axis. The quantities ρ and p depends on t only. Now, the field equations (2), (3) & (5) [with zero cosmological 'constant'] for the metric (7), with the help of (39) & (40) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{A_4\phi_4}{A\phi} + \frac{B_4\phi_4}{B\phi} + \frac{C_4\phi_4}{C\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\rho \tag{41}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \frac{A_4\phi_4}{A\phi} - \frac{B_4\phi_4}{B\phi} - \frac{C_4\phi_4}{C\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\rho \tag{42}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{A_4\phi_4}{A\phi} + \frac{B_4\phi_4}{B\phi} - \frac{C_4\phi_4}{C\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = -8\pi Gp \tag{43}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} + \frac{A_4\phi_4}{A\phi} + \frac{B_4\phi_4}{B\phi} + \frac{C_4\phi_4}{C\phi} - \frac{\phi_{44}}{\phi} + 3\left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\rho \tag{44}$$

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = -\rho\frac{G_4}{G} - (\rho + 3p)\frac{\phi_4}{\phi} \tag{45}$$

Equations (41) – (45) are a set of five independent equations in seven unknowns A, B, C, p, ρ, G and ϕ. Hence to get a determinate solution assume a relation between metric potentials given by (15). We also assume the equation of state

$$\rho = p \tag{46}$$

$\therefore$ The sets of equation (41) – (45), reduces to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{B_4\phi_4}{B\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\rho \tag{47}$$

$$2\frac{C_{44}}{C} + \left(\frac{C_4}{C}\right)^2 - \frac{B_4\phi_4}{B\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\rho \tag{48}$$

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{B_4\phi_4}{B\phi} + \frac{\phi_{44}}{\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = -8\pi Gp \tag{49}$$

$$2\frac{B_4C_4}{BC}+\left(\frac{C_4}{C}\right)^2+2\frac{C_4\phi_4}{C\phi}+\frac{B_4\phi_4}{B\phi}-\frac{\phi_{44}}{\phi}+3\left(\frac{\phi_4}{\phi}\right)^2=8\pi G\rho \tag{50}$$

$$\rho_4+2\rho\left(\frac{B_4}{B}+2\frac{C_4}{C}\right)=-\rho\frac{G_4}{G}-4\rho\frac{\phi_4}{\phi} \tag{51}$$

with the help of equation (47) & (49),

we get $p = -\rho$ or $\rho + p = 0$ (52)

which leads to domain wall models in the acceleratred universe dominated by a fluid with negative pressure such that the string energy condition is violated. However, since in the standard cosmology a fluid of negative pressure violating the string energy condition does not cluster at large scale in the relativistic regime equation (47) & (49) together yield

$$\rho = 0 = p \tag{53}$$

which shows that, stiff or self-gravitating domain wall do not survive in scale covariant theory of gravitation in this particular case.

3.2 Bianchi type -V space-time

We again, consider Bianchi type-V metric given by (25), in the comoving coordinate system we have from

$$T_1^1=T_2^2=\rho=T_4^4,\quad T_3^3=-p,\quad T_j^i\neq 0, \text{ for } i\neq j \tag{54}$$

Here pressure is taken in the direction of Z-axis. The quantities ρ & p depends on t only. Now, the field equations (2), (3) & (5) [with zero cosmological 'constant'] for the metric (25), with the help of (39) & (54) can be written as

$$2\frac{B_{44}}{B}+\left(\frac{B_4}{B}\right)^2-\frac{1}{A^2}-\frac{A_4\phi_4}{A\phi}+2\frac{B_4\phi_4}{B\phi}+\frac{\phi_{44}}{\phi}+\left(\frac{\phi_4}{\phi}\right)^2=-8\pi G\rho \tag{55}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{1}{A^2}+\frac{A_4\phi_4}{A\phi}+\frac{\phi_{44}}{\phi}-\left(\frac{\phi_4}{\phi}\right)^2=-8\pi G\rho \tag{56}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{1}{A^2}+\frac{A_4\phi_4}{A\phi}+\frac{\phi_{44}}{\phi}+\left(\frac{\phi_4}{\phi}\right)^2=8\pi Gp \tag{57}$$

$$2\frac{A_4B_4}{AB}+\left(\frac{B_4}{B}\right)^2-\frac{3}{A^2}=-8\pi G\rho \tag{58}$$

$$\frac{B_4}{B}-\frac{A_4}{A}=0 \tag{59}$$

$$\rho_4 + 3(\rho + p)\frac{B_4}{B} = -\rho\frac{G_4}{G} - (\rho + 3p)\frac{\phi_4}{\phi} \tag{60}$$

Equations (55) – (58) & (60) are set of five independent equations in seven unknowns A , B ,C , p , ρ , G and ϕ . Hence to get a determinate solution assume a relation between metric potentials (A=B) from equation (59). We also assume the equation of state

$$\rho = p \tag{61}$$

$\therefore$ The sets of equation (55) – (60), reduces to

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{1}{B^2} + \frac{B_4\phi_4}{B\phi} + \frac{\phi_{44}}{\phi} + \left(\frac{\phi_4}{\phi}\right)^2 = -8\pi G\rho \tag{62}$$

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{1}{B^2} + \frac{B_4\phi_4}{B\phi} + \frac{\phi_{44}}{\phi} + \left(\frac{\phi_4}{\phi}\right)^2 = 8\pi Gp \tag{63}$$

$$3\left(\frac{B_4}{B}\right)^2 - \frac{3}{B^2} = -8\pi G\rho \tag{64}$$

$$\rho_4 + 6\rho\frac{B_4}{B} = -\rho\frac{G_4}{G} + 4\rho\frac{\phi_4}{\phi} \tag{65}$$

with the help of equation (62) & (63),

we get $-\rho = p$ i.e. $\rho + p = 0$

by using equation of state $\rho = p$

$$\rho = 0 = p \tag{66}$$

which shows that, stiff or self-gravitating domain walls do not survive in scale covariant theory of gravitation.

4. Conclusion :

We have shown that Bianchi type -I and type-V cosmic strings models which represent Nambu string (geometric string), p-string & Reddy string do not survive in scale covariant theory of gravitation formulated by Canuto et al (1977) when we assume a relation between metric co-efficients.

We have also shown, in this particular case, that self-gravitating or stiff domain walls do not exists.

References :

1. Brans, C.H., Dicke, R.H. : Phys. Rev. **124**, 925 (1961)
2. Nordvedt, K.Jr. : Astrophys. J. **161**, 1069 (1970)
3. Saez, D., Ballester, V.J. : Phys.Lett. **A 1113**, 467 (1985)
4. Canuto, V.M., Hsieh, S.H., Adams, P.J. : Phys.Rev. Lett. **39**,429 (1977 a)
5. Canuto, V.M., Hsieh, S.H., Tsiang, E. : Phys.Rev. **D 16**, 1643 (1977 b)
6. Beesham, A. : Class. Quantum Gravity **3, 481** (1986 a)
7. Beesham, A. : Astrophys. Space Sci.,**119**, 415 (1986 b)
8. Beesham, A. : Class. Quantum Gravity **3**, 1027 (1986 c)
9. Reddy D.R.K. : Astrophys. Spa. Sci. **236**, 365-371 (2003)
10. Reddy, D.R.K., Venkateswarlu, R. : Astrophys. Space Sci., **136**,191 (1987)
11. Reddy D.R.K., Murth, Ch. S.V.V.R., Venkateswarly, R. : J. Ind. Math. Soc.**68**, 179 (2002)
12. Reddy D.R.K., Venkateswarlu, R. : Astrophys. Spa. Sci., **289**, 1 (2004)
13. Kibble, T.W.B. : J.Phys., **A9**, 1387 (1976)
14. Vilenkin, A. : Phys. Rev. **D 23**, 852 (1981)
15. Gott J.R. : Astrophys J. **288,** 422 (1985)
16. Letelier, P.S. : Phys. Rev. **D.28**, 2414 (1983)
17. Satchel, J. : Phys. Rev. **D.21**, 2171 (1980)
18. Krori, K.D., Chaudhari T., Mahanta, C.R., Majumdar, A. : Gen.Rel. Gravit. **22**, 123 (1990)
19. Banerjee, A., Samyal, A.K., Chakraborti, S. : Pramana J. Phys. **34**, 1 (1990)
20. Tikekar R., Patel L.K. : Gen.Rel. Gravit. **26**,647 (1990)
21. Tikekar R., Patel L.K. , Dadhich, N. : Gen. Rel. Gravit. (1994)
22. Rahaman, F., Chakraborti, S., Hossain, M., Begum, N., Bera J. : Ind, J.Phys. **B 76**, 747 (2002)
23. Reddy D.R.K. : Astrophys. Space Sci. **286**, 359 (2003 a)
24. Reddy, D.R.K. : Astrophys. Space Sci. **286**, 365 (2003 b)
25. Reddy, D.R.K. : Astrophys. Space Sci**. 300**, 381 (2005 a)
26. Reddy D.R.K. : J.Dyn. Syst. Theor. **3**,197 (2005 b)
27. Isper, J., Sikivie, P. : Phys. Rev. **D. 30**, 712 (1984)
28. Widrow, L.M. : Phys. Rev. **D.39**, 3571 (1989)
29. Goetz, G. : J.Math. Phys **31**, 2683 (1990)
30. Mukherji, M. : Class. Quan. Grav. **10**,131 (1993)

31. Wang, A. : Mod. Phys. Lett. **39**,3605 (1994)
32. Rahaman, F., Chakraborty, S., Kalam, M. : Int. J.Mod. Phys. **D 10**, 735 (2001)
33. Rahaman F. : Astrophys. Space Sci. **282**, 625 (2002)
34. Reddy, D.R.K. Rao, M.V.S. : Astrophys. Space Sci. **302**, 157 (2006)
35. Hillet, C.T., Schram, D.N., Fra, J.N. : Nucl. Part. Phys. **19**,25 (1989)
36. Adhav, K.S., Nimkar A.S., Dawande, M.V. : Astrophys. Space Sci. DOI 10.1007/s 10509-007-9506-8 (2007)
37. Adhav, K.S., Nimkar A.S., Holey, R.P. : Int. J. Theory Phys. DOI 10.1007/s /10773 – 007- 9357-7 (2007).