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String Cosmological Models with Magnetic Field

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Abstract A string cosmological models with magnetic field along X and *Z*-axis are investigated in the context of Ruban's space time. To obtain a determinant solution, equation of state for string model and relation between metric potential is considered. The physical and geometrical aspects are also discussed.

Keywords: *Ruban's metric, cosmic string, magnetic field*

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1. Introduction

String theory is a developing branch of theoretical physics that combines quantum mechanics and general relativity into a quantum theory of gravity. The strings of string theory are one-dimensional oscillating lines, but they are no longer considered fundamental to the theory, which can be formulated in terms of points or surfaces too.

In the last few years the study of cosmic string has attracted considerable interest as they are believed to play an important role in early stages of the universe. The gravitational effects of cosmic strings have been extensively discussed by Vilenkin [1], Gott [2], Letelier [3], Stachel [4] in general relativity. Relativistic string models in the context of Bianchi space times have been obtained by Krori *et. al.* [5], Bhattacharjee and Baruach [6] studied the problem of cosmic strings taking Bianchi type cosmologies with a self-interacting scalar field.

Also it is well known that the magnetic field significant role at the cosmological scale and is present in galactic and intergalactic spaces. The importance of the magnetic field for various astrophysical phenomena has been studied by many researchers. Melvin [7] has pointed out that during the evolution of the universe, the matter is in a highly ionized state and is smoothly coupled with the field, subsequently forming neutral matter as a result of universe expansion.

Jacobs [8,9], Collins [10], Roy *et.al.* [11] have investigated magnetized cosmological models for perfect fluid distribution in general relativity. Misra *et.al.* [12] have obtained the analogous relation between some components of metric and electromagnetic potentials for source free Einstein–Maxwell field described by Einstein-Rosen metric. Recently Adhav *et.al.* [13] have studied Bianchi Type III magnetized wet dark fluid cosmological model in general relativity. Sahoo *et.al.* [14] studied cylindrically symmetric cosmic strings coupled with Maxwell fields in bimetric relativity.

The purpose of the present work is to obtain Ruban's cosmological models in presence of cosmic string with magnetic field. Our paper is organized as follows. In section 2, we derive the metric and field equations. section 3 is mainly concerned with the physical and kinematical properties of the model. The last section contains some conclusion.

2. The Metric and Field Equations

We consider the space- time of Ruban's [15] in the form

$$ds^2 = dt^2 - Q^2(x,t)dx^2 - R^2(t)(dy^2 + h^2 dz^2) \tag{1}$$

where $h(y) = \dfrac{\sin\sqrt{k}\,y}{\sqrt{k}} = \begin{cases} \sin y & if \quad k = 1 \\ y & if \quad k = 0 \\ \sinh y & if \quad k = -1 \end{cases}$.

and k is the curvature parameter of the homogeneous 2-spaces t and x constants. The functions Q and R are free and will be determined.

The field equations for cosmic strings with magnetic field is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} \tag{2}$$

Where T_{ij} is effective energy momentum tensor of the cosmic strings in presence of magnetic field may be define as

$$T_{ij} = \rho\, u_i u_j - \lambda\, x_i x_j + E_{ij} \tag{3}$$

with

$$u^i u_i = -x^i x_i = -1, \tag{4}$$

and

$$u^i x_i = 0. \tag{5}$$

Here ρ is the rest energy density of cloud of strings with particles attached to them and λ the tension density of the system of strings. As pointed out Letelier [3] λ may be positive or negative, u^i describes the system four-velocity and x^i represents a direction of anisotropy, i.e. the direction of the strings.

We consider,

$$\rho = \rho_p + \lambda \tag{6}$$

Where ρ_p is the rest energy density of the particles attached to the string. Here ρ and λ are the functions of t only.

The energy momentum tensor of the electromagnetic field

$$E_{ij} = \frac{1}{4\pi}\left[-F_{is}F_{jp}g^{sp} + \frac{1}{4}g_{ij}F_{sp}F^{sp}\right]. \tag{7}$$

Case I: In this case, we have obtain cosmological model corresponding to cosmic string with magnetic field along X-axis

We assume that F_{23} is the only non-vanishing components of F_{ij}.

The Maxwell's equations is

$$\frac{\partial}{\partial x^j}\left(F^{ij}\sqrt{-g}\right) = 0. \tag{8}$$

Which Leads to

$$\frac{\partial}{\partial z}\left(F^{23}QR^2h\right) = 0. \tag{9}$$

Thus $F_{23} = H$, where H is constant.

We take F_{23} as the only non-vanishing of F_{ij} because a cosmological model which contains a global magnetic field is necessary anisotropic since the magnetic field vector is prepared specifies a preferred special direction. We assume that the current is flowing along the X-axis, so magnetic field is in the ZY-plane. Thus F_{23} is the only non-vanishing component.

The field equations (2) for the metric (1) with the help of (3-9) reduce to

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \lambda + \frac{1}{8\pi}\frac{H^2}{R^4h^2} \tag{10}$$

$$\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{R}}{R} + \frac{\ddot{Q}}{Q} = -\frac{1}{8\pi}\frac{H^2}{R^4h^2} \tag{11}$$

$$2\frac{\dot{R}\dot{Q}}{RQ} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \rho + \frac{1}{8\pi}\frac{H^2}{R^4h^2}. \tag{12}$$

Here over head dot represent partial differentiation with respect to t

To get a determinate solution one has to assume a physical or mathematical condition. In the literature Letelier [3], we have equation of state for the string model

$$\rho = \lambda\left(\text{Geometric string or Nambu string}\right) \tag{13}$$

By using (13) and relation between metric potential

$$Q = x^n R^n \tag{14}$$

The solution of field equations (10-12), we have

$$R = M(k_1t + k_2)^{\frac{1}{1-n}}$$

and

$$Q = N(k_1t + k_2)^{\frac{n}{1-n}}, \tag{15}$$

where $M = (1-n)^{\frac{1}{1-n}}$ and $N = x^n M^n$.

$$\rho = \lambda = \frac{(n+3)k_1^2 n}{(1-n)^2}\frac{1}{(kt_1 + k_2)^2} + \left(n^2+1\right)\frac{k_1^2}{(1-n)^2}\frac{1}{(k_1t+k_2)^2} + \frac{k}{M\left(k_1t+k_2\right)^{\frac{2}{1-n}}} \tag{16}$$

$$H^2 = -h^2k_1^2M^4\left(2n^2+n\right)\left(k_1t+k_2\right)^{\frac{2(n+1)}{1-n}}.$$

With the help of (15) the Metric (1) becomes

$$ds^2 = dt^2 - M^2(k_1t+k_2)^{\frac{2}{1-n}}dx^2 - N^2(k_1t+k_2)^{\frac{2n}{1-n}}(dy^2 + h^2dz^2).$$

Through a proper choice of coordinates and constants the model can be written as

$$ds^2 = \frac{dT^2}{k_1^2} - M^2(T)^{\frac{2}{1-n}}dx^2 - N^2(T)^{\frac{2n}{1-n}}(dy^2 + h^2dz^2). \tag{17}$$

Case II:

In this case, we have obtain cosmological model corresponding to cosmic string with magnetic field along Z-axis.

The field equations (2) for the metric (1) with the help of (3-7) reduce to

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = -\frac{1}{8\pi}\frac{H^2}{Q^2R^2} \tag{18}$$

$$\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{R}}{R} + \frac{\ddot{Q}}{Q} = \frac{1}{8\pi}\frac{H^2}{Q^2R^2} \tag{19}$$

$$\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{R}}{R} + \frac{\ddot{Q}}{Q} = \lambda + \frac{1}{8\pi}\frac{H^2}{Q^2R^2} \tag{20}$$

$$2\frac{\dot{R}\dot{Q}}{RQ} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \rho + \frac{1}{8\pi}\frac{H^2}{Q^2R^2}. \tag{21}$$

After solving equations (18-21), we have

$$R = M_2(k_3t + k_4)^{\frac{1}{M_1+1}} \tag{22}$$

$$\text{and } Q = M_3(k_3t + k_4)^{\frac{n}{m_1+1}},$$

where $M_2 = (M_1 + 1)^{\frac{1}{M_1+1}}$ and $M_3 = x^n m_2^n$

$$\rho = \lambda = 0 \tag{23}$$

$$H^2 = 8\pi M_2{}^2 M_3{}^2 M_4 \left(k_3t + k_4\right)^{\frac{2n+2}{M_1+1}}.$$

With the help of equations (22) the Metric (1) becomes

$$ds^2 = dt^2 - M_2{}^2 \left(k_3t + k_4\right)^{\frac{2}{M_1+1}} dx^2$$
$$- M_3{}^2 (k_3t + k_4)^{\frac{2n}{m_1+1}} (dy^2 + h^2dz^2).$$

Through a proper choice of coordinates and constants the model can be written as

$$ds^2 = \frac{dT^2}{k_3{}^2} - M_2{}^2 \left(T\right)^{\frac{2}{M_1+1}} dx^2$$
$$- M_3{}^2 (T)^{\frac{2n}{m_1+1}} (dy^2 + h^2dz^2). \tag{24}$$

3. The Physical and Kinematical Property

The physical quantities that are important in cosmology are spatial Volume, Scalar Expansion and Shear Scalar and have the following expression for the model given by (17).

Spatial volume

$$V = x^n M^{n+2} h\left(T\right)^{\frac{n+2}{1-n}} \tag{25}$$

Scalar Expansion,

$$\theta = \frac{k_1(n+2)}{1-n}\frac{1}{T} \tag{26}$$

Shear Scalar,

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{6}\left[\frac{k_1(n+2}{(1-n)}\right]\frac{1}{T^2}. \tag{27}$$

The expression for the tension density and string density for cosmic string with magnetic field along X-axis are given by equation (16) whereas along Z-axis are given by (23) respectively.

It may be observed that at initial moment when $T=0$. The special volume will be zero. While string density and tension density diverges. When $T \to o$, then scalar expansion and shear scalar tends to ∞.

For large value of T ($T \to \infty$) we observed that scalar expansion and shear scalar becomes zero.

Similar results are obtained for model (24) except string density and tension density.

4. Conclusion

Basically our motivation to investigate string cosmological model with magnetic field along different axes in general theory of relativity. String cosmology with magnetic field and space times associated with them have cosmological interest due to their important application in structure formation of the universe. Here we have presented string cosmological models with magnetic field along different direction for Ruban's background. The models in both the cases are similar. Along the X-axis, tension density and string density of cosmic string are present but along Z-axis tension density and string density of cosmic string are absent. Therefore in a Ruban's background only cosmological model with magnetic field are presents along X-axis.

The models are free from initial singularities and they are expanding, shearing and non-rotating in the standard way.

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